

## DOCTORAL DISSERTATION ABSTRACT

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Scientific discipline: mathematics

Title of the doctoral dissertation: Periodic Solutions of Nonlinear Damped Wave Equations on the Whole Euclidean Space

Doctoral dissertation abstract: The dissertation presents criteria for the existence of periodic solutions of nonlinear damped wave equations with a Kato-Rellich type potential, and a time-periodic nonlinear term. In the resonant case at infinity, i.e., when the nonlinear term is bounded and the kernel of the linearization of the right-hand side at infinity is nontrivial, the existence of periodic solutions is shown assuming Landesman-Lazer type conditions or sign conditions. In the non-resonant case, i.e., when the kernel of the linearization of the right-hand side at infinity is trivial, the existence of (nontrivial) periodic solutions is established provided the total multiplicities of the negative eigenvalues of the linearizations at zero and at infinity are different modulo 2. The approach involves finding fixed points of the translation along trajectories operator using the index theory for contractions with respect to the Hausdorff measure of non-compactness.

Chapter 1 is a preparation for the application of the method of translation. The main result establishes the continuous dependence of solutions of a sequence of evolutionary problems, assuming integral convergence of the nonlinear terms. From this result, a general averaging principle is obtained, which is later used in the proof of the non-resonant averaging principle.

In Chapter 2, we deal with the linear wave equation. We prove that, under appropriate assumptions on the potential at infinity, the essential spectrum of the Schrödinger operator is a subset of the positive half-line. We show that the wave operator generates a semigroup of bounded linear operators and determine its spectrum. We define an inner product dependent on the potential and a suitably chosen parameter. We prove that this product induces a norm equivalent to the standard one on the phase space. We show that the semigroup is contractive with respect to the new norm on a subspace of finite codimension and that it is a contraction with respect to the Hausdorff measure of non-compactness on the entire phase space. We determine the spectrum of the semigroup restricted to the specific subspaces and compute the fixed-point index of the translation operator associated with the linear wave equation.



In Chapter 3, we obtain an estimate for the Hausdorff measure of non-compactness for sequences of nonlinearities and associated Nemytskii operators. We show that the translation operator associated with a family of evolutionary wave equations is a contraction with respect to the Hausdorff measure of non-compactness. We consider a sequence of changing frequencies evolutionary problems and the corresponding sequence of periodic solutions. We then prove that the sequence of initial conditions is relatively compact in the phase space, which is used in the non-resonant averaging principle.

Chapters 4 and 5 contain the main results of the dissertation. In the resonant case (Chapter 4), we compute the fixed-point index of the translation operator associated with a family of autonomous wave equations, and then the index of the translation associated with the original equation with a parameter (the resonant averaging principle), and derive the continuation principle. We prove geometric conditions concerning the Nemytskii operator. We present the proof of the criterion for the existence of periodic solutions and then its application to the equation with the Coulomb potential and suitably chosen nonlinearities.

In Chapter 5, we prove the non-resonant averaging principle for a family of wave equations, the non-resonant averaging principle for the index, and the continuation principle. Next, we prove sufficient conditions for *a priori* estimates. We compute the fixed-point index of the translation operator associated with the autonomous wave equation. We present the proof of the criterion for the existence of periodic solutions and its application to the equation with the Coulomb potential and appropriate classes of nonlinearities.

*Władysław Klinikowski*

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