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Optimal Investment Strategies and Risk Sharing in a Hybrid Pension Scheme

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Introduction

Motivation

The urgent need for sustainable pension systems is increasingly recognized as populations worldwide grow older and life expectancies rise. This study is driven by the global challenge of ensuring that pension systems remain robust in the face of demographic shifts, such as longer life spans and declining birth rates. These issues raise significant concerns about the long-term viability and fairness of pension distributions across generations (S. Wang & Lu, 2019). As global demographics shift towards older populations and life expectancies increase, traditional pension systems face mounting pressures that threaten their sustainability and efficacy. Defined Benefit (DB) and Defined Contribution (DC) plans are the primary pension schemes, each with distinct advantages and limitations. DB plans offer predictable benefits but place substantial financial risk on plan sponsors, often leading to unfunded liabilities, especially in volatile markets (Cocco & Lopes, 2011). Conversely, DC plans mitigate employer risk by shifting investment and longevity risks to the individual, potentially resulting in inadequate retirement incomes due to market downturns or poor investment decisions (Poterba et al., 2007). This dichotomy presents a critical need for innovative pension designs that combine the strengths of both systems while mitigating their weaknesses.

The hybrid pension scheme, a concept that integrates elements of both DB and DC plans, emerges as a promising solution to address these challenges. Hybrid plans aim to distribute risks more equitably between sponsors and beneficiaries, potentially enhancing stability and predictiveness for retirees while controlling the financial exposure of employers. This dissertation explores applying and optimizing hybrid pension schemes as a strategic response to the demographic and economic challenges impacting pension sustainability worldwide, drawing on the experiences and reforms observed in distinct national contexts like China and Poland (Guan & Liang, 2014; He et al., 2022). China and Poland currently employ DC schemes, and this dissertation proposes how hybrid schemes could be adapted and introduced in these settings. By using examples from these two countries, the dissertation illustrates how a hybrid scheme could offer improvements for pensioners over the traditional DC model. This represents one of the key innovations of this research.

In this dissertation, we choose China and Poland as focal points for comparative analysis, which is motivated by their shared and distinct characteristics and provides a comprehensive spectrum for examining the efficacy of hybrid pension schemes. Both countries, recognized as developing economies with high economic growth, have experienced significant demographic shifts, particularly from the baby boom of the 20th century, whose retirees are now placing increasing pressures on pension systems. Each has responded with similar pension reforms, notably adopting Defined Contribution (DC) schemes, yet they operate within very different demographic and economic environments. With its vast population, China faces unique challenges due to its rapidly aging demographic, exacerbated by past one-child policies and increased life expectancy. Poland, transitioning from a post-communist structure to a diversified market, confronts the legacy of its historical context while integrating modern social security measures.

This comparative study is further enriched by the contrasts between the two: while both countries implement DC schemes, the specifics of their pension plans—such as contribution rates, benefits, and investment strategies—differ markedly. These differences offer valuable insights into how pension systems can be tailored to meet specific national needs while addressing common challenges such as sustainability and the management of aging populations. By analyzing these two diverse systems, the dissertation aims to uncover how varied demographic trends and policy frameworks influence the design and success of pension reforms, providing lessons on managing pension schemes in diverse socio-economic contexts.

Research Problem

This dissertation addresses a critical gap in pension management: the need for systems that adeptly balance risk, ensure sustainability, and maintain intergenerational equity amid aging populations. The innovative contribution of this research is twofold. Firstly, it enhances the existing concept of hybrid pension schemes, which combine the stability of Defined Benefit (DB) plans with the flexibility of Defined Contribution (DC) plans. This approach offers a refined solution to mitigate the inherent limitations of traditional pension models. Secondly, the study integrates a sophisticated population dynamics model, leveraging actual demographic data from China and Poland to calibrate the pension schemes. This method allows for a nuanced analysis of how hybrid pension plans can be tailored and optimized to meet the specific challenges and demographic realities of different national contexts. By embedding real-world demographic structures into the pension model, this research illuminates the potential of hybrid schemes to provide resilient frameworks that can effectively navigate the financial uncertainties and demographic shifts characteristic of the 21st century, making a significant contribution to the field of pension economics.

Aim

The primary aim of this dissertation is to evaluate the effectiveness of hybrid pension schemes in managing demographic risks and maintaining financial sustainability in the face of aging populations.

The general aim was decomposed into detailed ones as follows:

1. To determine the impact of birth and mortality rates on long-run consumption under the hybrid pension scheme.
2. To compare the efficiency of traditional DB and DC pension schemes with the hybrid pension scheme.
3. To compare the results obtained for China and Poland from the demographic risk perspective.
4. To check the results for robustness by changing the key parameters.
5. To provide the pension policy recommendations for China and Poland.

Hypotheses:

H1: There is greater stability and flexibility in hybrid pension schemes, which combine features of DB and

DC plans, compared to traditional pension models when confronted with demographic shifts and economic uncertainties.

H2: There is superior performance in terms of risk management and sustainability in hybrid pension schemes, particularly in contexts characterized by varying birth rates and aging populations, compared to standalone DB and DC schemes.

H3: There are significant differences in the suitability of pension models across different national contexts, influenced by unique demographic structures and policy frameworks.

Methodology

In this dissertation, we employ a multi-faceted methodology that integrates demographic analysis, financial modeling, and advanced computational techniques to address the complexities inherent in managing pension schemes. Central to our demographic analysis is the Lee-Carter model, renowned for its robustness in forecasting mortality rates. This model breaks down mortality rates into age-specific and time-specific components, allowing for a detailed exploration of shifts in life expectancies crucial for pension fund management. We refine the traditional Lee-Carter model with geometric and hyperbolic adjustments to more precisely capture contemporary trends in mortality, thus aligning it with the long-term demographic changes that significantly affect pension systems.

The Overlapping Generations (OLG) model is used to simulate financial interactions within the pension framework by modeling the economic lifecycle of individuals across multiple generations who live, work, and retire at different times. This model divides the population into distinct age cohorts, each at different life stages—actively working or retired—thereby facilitating analysis of economic activities and financial needs through various life phases. It provides insights into intergenerational equity by demonstrating how pension costs and benefits are distributed among different generations, ensuring that no single group bears excessive financial burdens. Savings and investment behaviors are also simulated, highlighting how individuals' economic decisions influenced by pension policies impact the overall system's stability and sustainability. This is supplemented by financial market simulations that incorporate both risk-free and risky assets to examine different investment strategies and their impact on the sustainability and risk profiles of pension funds while incorporating key demographic changes.

A pivotal element of our methodology involves the integration of Monte Carlo simulations with Bayesian Optimization. Monte Carlo simulations provide a robust mechanism for analyzing the variability and uncertainty of pension outcomes under stochastic economic and demographic scenarios. Bayesian Optimization is leveraged to fine-tune the parameters of the pension scheme, with a focus on maximizing participant welfare as quantified by Certainty Equivalent Consumption (CEC). This step is critical to ensure that the proposed pension models are not only theoretically robust but also practically feasible.

Additionally, the methodology includes a detailed calibration of the pension models based on actual demographic data from China and Poland. This calibration allows for a nuanced analysis of how hybrid pension plans can be tailored and optimized to meet the specific challenges and demographic realities of different national contexts. By embedding real-world demographic structures into the pension model, this research illuminates the potential of hybrid schemes to offer resilient frameworks that can effectively navigate financial uncertainties and demographic shifts.

The entire computational process is implemented using Python, facilitating the intricate modeling of demographic scenarios, financial simulations, and the application of optimization algorithms. Python's capabilities enable sophisticated data manipulation and complex statistical analysis, enhancing the precision and depth of the study. By amalgamating these diverse methodologies, this dissertation aims to provide a comprehensive analysis of how hybrid pension schemes can be optimized to manage demographic risks and financial uncertainties effectively. The results are intended to make a significant scholarly contribution to the field of pension economics and offer actionable insights for policymakers and industry practitioners.

The empirical study is projected to follow the four-stages scheme. It comprises of Data Collection and Preparation, gathering essential demographic and economic data from China and Poland; Model Calibration and Validation, adjusting and validating the Lee-Carter model with this data; Simulation and Analysis, using calibrated OLG models to conduct simulations via Monte Carlo methods and Bayesian Optimization, assessing impacts on hybrid pension schemes; and Policy Formulation and Recommendations, translating findings into tailored policy recommendations for each country, focusing on optimizing pension parameters for sustainability and fairness.

Thesis Structure

The thesis consists of the following parts: introduction, five chapters, conclusion, and policy recommendations. It is completed with seven appendixes.

Introduction: Outlines the dissertation's motivation, aims, hypotheses, and methodology.

Chapter 1 - Characteristics of Pension Schemes: Discusses the structure and types of pension schemes, with a detailed review of existing literature on pensions in China and Poland, and presents factual data on their pension systems.

Chapter 2 - Population Dynamics: Analyzes demographic trends and their implications for pension systems, employing mortality rate models to project future demographics.

Chapter 3 - Hybrid Pension Scheme Potential for Risk Reduction: Evaluates the potential of hybrid pension schemes to reduce risk and enhance intergenerational equity.

Chapter 4 - Optimal Investment Strategies-China Simulation: Applies theoretical models to simulate and analyze the performance of various pension schemes in China's specific context.

Chapter 5 - Optimal Investment Strategies- Poland Simulation: This is similar to Chapter 4, but it focuses on Poland, providing comparative insights between the two countries.

Conclusion and Policy Recommendations: Summarize key findings, discuss policy implications, and suggest areas for further research.

Appendixes 1-7: Provide partial Python codes for implementing the Lee-Carter model adaptations, demographic forecasting, and simulations of hybrid pension schemes, as discussed in this dissertation. I personally authored the Python code provided for this dissertation.

Chapter 1. Characteristics of Pension Schemes

1.1. Introduction

This chapter aims to provide a comprehensive overview of pension schemes, focusing on the basic framework, types, and characteristics of various pension plans. It will also delve into the existing literature on pension schemes in China and Poland, offering valuable insights and analysis of the current state of pension systems in these countries. The chapter will then present an in-depth examination of the three pillars of pension schemes in both China and Poland, highlighting their distinct features, structures, and contributions to the overall pension landscape. Through this exploration, the chapter will establish a solid foundation for understanding the complexities and nuances of pension schemes, as well as the challenges and opportunities they present for the future of retirement security.

1.2. Basic Pension Scheme Framework

The World Bank (1994) established a framework for pension obligations that included three pillars. Pillar I, II, and III pension schemes are a three-pillar framework that has been adopted by some countries to provide a comprehensive approach to retirement income security. The three-pillar framework is intended to provide a comprehensive approach to retirement income security, with Pillar I providing a basic level of retirement income, Pillar II providing additional retirement income through employer-sponsored plans, and Pillar III providing individuals with the flexibility to save and invest for retirement on their own. The specific design and implementation of each pillar may vary by country.

Pillar I is the first pillar and is also known as the basic or mandatory pension scheme. It is typically a government-run social security system funded through taxes or contributions from employers and employees. This pillar provides a basic level of retirement income to all eligible citizens or residents, regardless of their income or employment status. Pillar II is the second pillar and is also known as the occupational or company pension scheme. It is usually a privately managed, employer-sponsored retirement plan that is designed to supplement the basic pension provided by Pillar I. Contributions to Pillar II plans are often made by both employers and employees, and the benefits are based on the individual's salary and length of service. Pillar III is the third pillar and is also known as the voluntary or personal pension scheme (Koutronas & Yew, 2017). It is typically a privately managed retirement plan that individuals can contribute to voluntarily, either on their own or through their employer. The benefits of Pillar III plans are based on the individual's contributions and investment returns.

In summary, the World Bank's three-pillar framework for pension obligations, established in 1994, has been adopted by countries like Poland and China to provide a comprehensive approach to retirement income security. Pillar I encompasses government-run social security systems offering basic retirement income, while Pillar II involves employer-sponsored plans that supplement Pillar I. Lastly, Pillar III represents voluntary, privately managed retirement plans that rely on individual contributions and investment returns. The specific design and implementation of each pillar may differ between countries, yet both Poland and China have utilized this framework to create a holistic pension scheme that addresses the retirement income needs of their citizens.

1.3. Basic Pension Types

Defined Benefit (DB) and Defined Contribution (DC) pension plans are traditional types of pension plan designs commonly implemented in various countries. DB pension plans, which guarantee a predetermined benefit after retirement, are prevalent in countries like the United States and the United Kingdom. On the other hand, DC pension plans, where both the employee and employer contribute funds to individual accounts, are widely used in countries such as China, Poland, Australia, and Germany. In addition to these traditional pension plans, some countries have adopted Hybrid Pension Plans that combine elements of both DB and DC plans, offering unique structures and risk allocations. These hybrid plans can be found in countries like the Netherlands, Sweden, Canada, and the United States.

1.3.1. Defined Benefit Pension Plan

Defined Benefit (DB) pension schemes provide a guaranteed benefit upon retirement, with the amount determined by a predefined formula linked to factors such as an employee's salary and years of service (Poterba et al., 2007). Common examples of DB plans include "career average" and "final salary" plans. In these plans, participants do not have individual investment accounts; instead, the plan sponsor (employer) centrally manages collective investments. The sponsor bears the risks associated with investments, longevity, inflation, interest rates, and political decisions and must maintain contributions to cover all promised benefits. As a result, employees receive predetermined benefits each month after retirement, mitigating the uncertainty of their post-retirement income.

In a DB pension, employers and employees bear different risks. For employers, the most significant risks are investment risk and longevity risk. When the investment market performs poorly and the return on assets is lower than expected, then the pension is underfunded. This is when employers have to increase contributions to enable retirees to receive guaranteed benefit payments. Longevity risk refers to the risk that retirees live longer than expected, which means that the period of benefit payment increases (Cocco & Lopes, 2011). This risk is pooled and borne by the employers. For employees, on the other hand, the main risk to which they are exposed is the risk of inflation. Inflation risk is the risk that inflation will reduce the real value of benefits accruing to DB plan participants. As mentioned earlier, benefit payments under the DB Plan are calculated based on the participant's salary. If inflation reduces the real value of the earnings base used to calculate these benefits, it will reduce the real value of the accrued benefits.

1.3.2. Defined Contribution Pension Plan

In a Defined Contribution (DC) pension plan, each employee holds an individual account to which both the employee and employer contribute. These contributions are typically based on a fixed percentage of monthly earnings, as outlined in the employment agreement. Throughout an employee's working career, contributions are made monthly and invested in the market (Sialm et al., 2015). Upon retirement, employees receive benefits accrued in their accounts. However, these benefits cannot be guaranteed in advance, as they depend on market conditions and other factors beyond the control of both the employer and employee. A key distinction between DC and DB plans lies in the portability of assets. When an employee leaves his/her job, they can transfer their DC plan assets to a new employer's plan or an individual retirement account, enabling flexibility in managing retirement savings across different employers (Ilmanen et al., 2017).

The risk distribution is different in the DC plan compared to the DB plan. Under the DC plan, the investment risk and longevity risk are suffered by the employees. The amount accrued in an employee's account at retirement constitutes the entirety of the retirement benefit. Therefore, if the funds are invested well, the employee receives a corresponding benefit, but if the investments perform poorly then the benefit suffers. In a DC plan, longevity risk is primarily borne by the employee himself or herself. This is because the amount of money they receive when they retire depends on how much money they have in their personal accounts. For the retiree, longevity risk may involve the loss of a financial resource, as the retirement benefit may not be sufficient to last him or her for the rest of his or her life (Yang & Huang, 2009).

1.3.3. Hybrid Pension Plan

Hybrid pension schemes combine elements of both DB and DC plans. For example, participants may have individual DC accounts for investing contributions, while benefits are calculated using a DB formula. These plans often distribute risks between sponsors and employees or allocate them among participants and retirees. He et al. (2022) suggest that, in a narrow definition, hybrid pension schemes should exhibit characteristics such as adjustable contribution and benefit rates to minimize unsustainable risk, with adjustments based on specific metrics. These funds are jointly managed, allowing for the equitable allocation of unsustainable risk across cohorts. Broadly defined, there are three typical hybrid pension schemes: the Collective Defined Contribution (CDC) plan in the Netherlands, the non-financial DC plan in Sweden, and the cash balance plan in the United States and Canada (He et al., 2022).

Collective Defined Contribution (CDC) plans are trust-based occupational pension plans structured within the framework of a DC plan. In these plans, employer and employee contributions are pooled and invested to provide each member with a target retirement income. Members possess a percentage share of the collective plan assets, as opposed to an individual share found in traditional DC plans (Sorsa & van der Zwan, 2022). Nevertheless, members can choose to roll over from the CDC plan to purchase their own annuity or withdraw personal funds. Investment pooling in CDC plans offers potential advantages such as improved overall returns, economies of scale, shared mortality risk, and the implementation of higher risk-return strategies. A key distinction between CDC and DB plans is that CDC plans provide fluctuating benefits based on the plan's assets, offering expected target benefits rather than guaranteed ones. Consequently, if a funding shortfall occurs, employers are not required to provide additional funding to maintain a specific level of benefits.

In Sweden, the Non-Financial Defined Contribution (NDC) plan involves allocating a large portion of pension contributions to a nominal account, with the remainder going to an individual account. Benefits become payable at age 61, and the account balance can be converted to a pension upon retirement. Benefit duration is adjusted based on the life expectancy of respective retiree cohorts (Holzmann et al., 2019). The pension system provides a guaranteed pension for individuals with little or no pension income, while the income pension serves to offset the means-tested guarantee pension. Lifetime contributions determine pension benefits under the NDC plan, with the benefit aspect being less prominent due to the focus on contributions (Auerbach & Lee, 2011). As life expectancy increases, annual benefits at a given retirement age decrease, necessitating individuals to work longer and save more to cover their retirement expenses (Holzmann et al., 2019)).

Cash balance plans are hybrid plans that combine elements of both defined benefit and defined contribution pension schemes. Although they are classified as defined benefit plans, they share certain characteristics with defined contribution plans. The cash balance plan has its origins in the US tax law changes of the early 1980s, which allowed employers to explore new pension plan designs. Benefits consultants and actuaries developed cash balance plans in the 1980s to address concerns regarding the volatility and cost of traditional defined benefit plans. The first cash balance plan was introduced by Bank of America in 1985 (Niehaus & Yu, 2005). In a cash balance plan, an employee's pension benefit accrues as a lump-sum amount known as the "cash balance." This balance comprises "pay-related credits" that are periodically added based on the employee's salary, age, and seniority. Additionally, "interest-related credits" are incorporated at an annually adjusted crediting rate, linked to either a market rate or the Consumer Price Index (CPI) (He et al., 2022).

1.4. Literature Review of Pension Scheme

1.4.1. Research on Pension Schemes

Recent literature on pension schemes has addressed several critical issues including savings behaviors, the interplay of demography, and sustainability within pension systems. One area of literature has focused on the intersection of savings behaviors and pension schemes, examining how different types of pension systems influence personal and national savings rates. Blau's (2016) study employs a dynamic life cycle model to demonstrate how DB and DC pensions influence personal savings through the "crowd out" effect. His findings indicate that while DB pensions modestly reduce personal savings, DC pensions have a more pronounced effect. Blau suggests that pensions complicate the direct assessment of their impact on savings due to their role in providing risk coverage alongside wealth accumulation. Building on the theme of pension impact on savings, Lachowska and Myck (2018) delve into the effects of Poland's 1999 public pension reform. Their analysis shows that reductions in public pension wealth correlate with increased private savings, especially among more educated households. This study underscores the adaptive behaviors of households in response to changes in pension wealth, suggesting that people may compensate for lower pension benefits by boosting their private savings. Further exploring the relationship between private pension systems and savings, Ertuğrul and Gebeşoğlu (2020) examine the long-term effects of private pensions on Turkey's national savings using an Autoregressive Distributed Lag (ARDL) model. Their research confirms a positive contribution of private pensions to national savings. However, they also highlight the complexity of this relationship due to potential substitution effects, where shifts in the form of savings might not lead to an overall increase in savings.

The impact of demographic changes on pension systems is also a critical focus in recent literature, emphasizing the challenges and responses to aging populations and changing fertility rates. Lacomba and Lagos (2006) explore how an aging demographic strains pay-as-you-go (PAYG) pension systems, suggesting solutions like delaying retirement age, tax increases, and benefit reductions to maintain system viability. Fenge and Scheubel (2017) link public pensions to declining fertility rates, showing that pensions reduce the economic incentive for having children, further influencing demographic patterns. McGrattan and Prescott (2017) propose transitioning from pay-as-you-go (PAYG) to a savings-based system in the U.S., predicting this would alleviate demographic pressures by reducing reliance on payroll taxes and enhancing welfare across generations. Finally, Beetsma et al. (2020) reveal that while demographic trends shape the direction of pension reforms, the timing of these reforms correlates more closely with economic cycles than previously understood. Together, these studies underscore the complex interactions between demographic shifts, economic conditions, and pension policy, highlighting the need for multifaceted strategies to address the evolving challenges in pension systems.

After addressing demographic changes in pension systems, the focus naturally shifts to sustainability, a critical concern that has been extensively explored in contemporary research. Blake and Mayhew (2006) scrutinize the sustainability of the UK's state pension system, highlighting the challenges posed by demographic shifts and declining fertility rates. They emphasize that the system's sustainability depends on factors such as economic activity rates across age groups, real wage growth, and state pension contribution rates. Interestingly, they discuss the potential of immigration to offset demographic shortfalls while cautioning against sustainability threats like long-term unemployment and early retirements, particularly among those aged 35-49. In Malaysia, Jaafar et al.(2019) examine the Employees' Provident Fund (EPF), pinpointing sustainability challenges and advocating for reforms like enhanced investment diversification and governance transparency. They suggest transitioning from an unfunded defined benefit scheme to a funded defined contribution scheme to boost domestic savings and develop national capital markets. Turning to the European context, Alonso-Alonso-Fernandez et al. (2018) introduce the Gender Adequacy Index (GAI) to assess pension adequacy with a focus on gender disparities. They propose a dynamic evaluation of pension adequacy, examining replacement rates both at retirement and during the post-retirement years. They also highlight the concept of the Sustainability of Adequacy, which addresses the need to adapt working lives and retirement ages in response to population aging. Finally, Hinrichs (2021) reviews recent pension reforms in Europe, underscoring the challenges posed by non-standard employment patterns and their implications for pension systems. Hinrichs points out the growing trend of flexible employment, warning that it may lead to increased income inequality and pension inadequacy due to the decreasing likelihood of long-term, continuous employment up to the retirement age.

In conclusion, the literature on pension schemes highlights interconnected themes, including the impact of pensions on savings behaviors, the demographic pressures on these systems, and the overarching challenges of sustainability. Research in this area delves into how different pension systems influence personal and national savings, underscoring complex impacts and interactions. Additionally, studies on demographic changes advocate for policy adjustments to sustain pensions amidst aging populations and declining fertility rates. Furthermore, discussions on sustainability emphasize the need for strategic reforms to enhance the resilience and adequacy of pension systems, ensuring their viability in the face of evolving economic and demographic landscapes.

1.4.2. Research on China

In recent years, China's pension system has garnered significant research interest due to ongoing reforms that aim to tackle the nation's aging population, economic disparities, and regional imbalances. A substantial body of scholarly work has identified considerable inequity within China's pension system, emphasizing the necessity for continued research to comprehend the factors contributing to these disparities. Wang et al., (2014) offer a thorough assessment of the equity within China's multifaceted pension system by calculating fairness coefficients for pensions across different schemes, considering income, contributions, demand, and generational gaps. Their findings disclose absolute unfairness in old-age pensions across various schemes, underscoring the substantial challenges in providing equitable pensions.

Shen et al. (2018) explore the effects of social welfare programs on inequality, revealing uneven distribution across generations and income quartiles. This research highlights the limitations of China's benevolent state in addressing inequality and emphasizes the need to understand the efficacy of social welfare programs in alleviating income inequality. Lu and Dandapani (2023) examine the pension gender gap in China and its causes, utilizing micro-empirical simulations of the country's employee pension benefits model. They pinpoint wage inequality, mandatory early retirement for women, and longer female life expectancy as primary contributing factors.

These pivotal studies contribute to our understanding of the inequities within China's pension system, shedding light on the factors contributing to disparities and offering valuable insights for future reforms. By examining the fairness of pension schemes, the impact of social welfare programs on inequality, and the pension gender gap, the research collectively helps identify areas for improvement in addressing economic disparities and fostering a more equitable pension system.

A multitude of researchers have explored diverse facets of pension system unification in China, aiming to create a fairer and more sustainable social security framework. Yang (2009) presents a thorough examination of China's urban public pension system, addressing concerns about replacement rates and population growth rate while employing an overlapping generations (OLG) model to elucidate the complex interplay between individual account benefits, social pool benefits, and consumption patterns. Yang (2014) extends the analysis by examining the potential for long-term financing between China's public pension system and public rental housing using the OLG model, offering vital insights into the feasibility of interlinking these sectors and valuable policy recommendations.

Wu et al. (2022) investigate the crucial issue of optimizing delayed retirement policies in China, taking into account the country's rapid population aging and its consequences for the Social Security Fund and retiree welfare. They also utilize the OLG model in their analysis. Ji et al.(2022) propose an innovative method for optimizing primary social security pension policies in urban China by integrating multi-stage stochastic programming (MSP) and dynamic stochastic control (DSC). This approach allows for an in-depth analysis of the effects of alternative contribution rates, demographic shifts, and interest rate policies on pension system solvency and liquidity. The authors showcase the effective management of pension fund liquidity and maintenance of system solvency through their hybrid MSP and DSC model.

Taken together, these pivotal studies provide valuable insights into the optimal design of pension systems in China, addressing the country's distinct demographic and economic challenges. By examining various aspects of pension policy, this research contributes to the development of efficient and sustainable solutions for China's aging population. As a result, these comprehensive investigations serve as an essential knowledge resource for policymakers, equipping them to make informed decisions for the future of pension systems in China.

1.4.3. Research on Poland

The Polish pension scheme has undergone numerous reforms, impacting various demographic groups and raising concerns about its sustainability and adequacy. Zajicek et al. (2007) investigate the consequences of these reforms, particularly focusing on age, gender, and class intersections. They argue that state actions unevenly distribute the economic costs across different groups, with class-advantaged, middle-aged, and young men emerging as the main beneficiaries, while older women, young women, and low-waged men bear the brunt of the changes. Consequently, they emphasize the importance of considering age relations, power dynamics, and social inequalities.

In addition, Égert (2013) delves into the implications of changes in Poland's second pension pillar for public finances. Employing a stylized model, Égert simulates the long-term effects of weakening the second pillar on fiscal sustainability and pension benefits. The findings suggest that, in the baseline scenario, the reduction of the second pillar would lead to a permanent decrease in future pension system debt. However, under pessimistic assumptions, the pension system's deficit and debt may rise above the levels observed in the no policy change scenario in the long run.

Lastly, Marcinkiewicz (2019) explores the relationship between the development of voluntary pension systems and the adequacy of mandatory pension systems across Europe, including Poland. The study proposes a Voluntary Pensions Index (VPI) for cross-country comparisons and finds that Poland has a poorly developed voluntary pension system due to the presence of mandatory private pensions. This finding supports the idea that mandatory private pensions may crowd out voluntary ones, and it implies that supplementary pensions play a greater role in Poland due to the limitations of the mandatory system. Overall, Marcinkiewicz's research highlights the need to consider the role of voluntary pensions in addressing the inadequacies of the mandatory system.

And there are some researchers looking for the optimal pension structure in Poland. Kurach and Papla (2016) delve into the optimal hedge ratio for mandatory pension funds, challenging conventional knowledge on currency hedging and emphasizing the importance of portfolio diversification. Their findings offer valuable guidelines for emerging market investors and can potentially be applied to pension funds in other countries.

Complementing this research, Kurach et al. (2019) explore the optimal portfolio structure for Poland's two-pillar mandatory pension system. Through simulations, the authors determine that the current fixed split of pension contributions between the non-financial (first) and financial (second) pillars is suboptimal. They advocate for a time-varying approach to improve allocation efficiency. This innovative study has implications not only for Poland's pension regulations but also for other countries facing similar challenges in their pension systems.

Adding to this literature, Tyrowicz et al. (2018) investigate the effects of aging and pension system reforms on consumption and wealth inequality in Poland. Employing an overlapping generations (OLG) model, the study highlights the complexity of Poland's pension system and underscores the importance of considering both endowment-related and preference-related sources of inequality when evaluating pension system reforms. The authors also examine the role of minimum pension benefits in partially counteracting inequality, shedding light on the fiscal costs of such interventions.

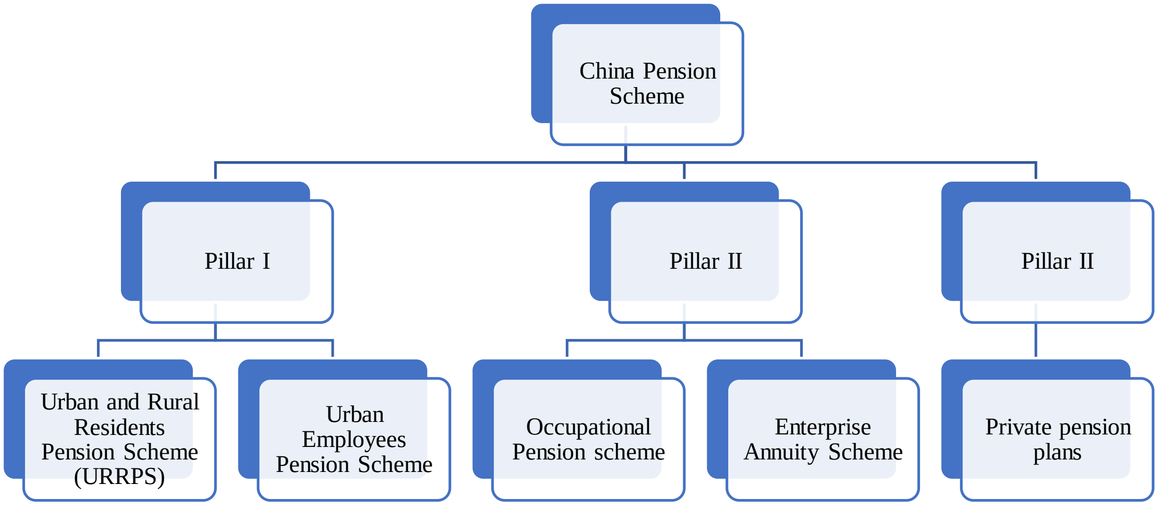
Building on this, Xie et al. (2023) provide a unique perspective on the sustainability of Poland's pension system by examining the saving behaviors of younger generations, specifically Gen Z and Gen Y. Their research reveals that personal responsibility for financial security in old age significantly influences retirement savings decisions among these young adults. This finding suggests that fostering a sense of personal responsibility for financial security in old age could be a key strategy for ensuring the sustainability of Poland's pension system. Furthermore, the authors highlight the role of trust in state institutions in influencing young adults' decisions to save for retirement, suggesting that maintaining and enhancing trust in state institutions could be another important factor in promoting the sustainability of the pension system.

Collectively, these studies provide valuable insights into the design and optimization of Poland's pension system. By analyzing various aspects, such as hedge ratios, portfolio structures, and the impact of aging and pension reforms, this body of research contributes to the ongoing debate on creating sustainable and equitable pension systems in Poland and beyond.

1.5. Facts of China Pension Scheme

Many developed countries have relatively stable pension systems, with no significant changes in the underlying system structure over the past few years. In China, however, the structure of the pension system continues to evolve and change, in large part to meet the challenges of an aging population (T. Liu & Sun, 2016). China’s pension scheme is made up of three pillars (see Figure 1.1). The first pillar is the public pension scheme. The public pension scheme is the basic pension scheme, which includes the Urban Employees’ Pension Scheme and the Rural and Urban Residents Pension Scheme. The second pillar is an important supplement to the basic pension scheme in China. It consists of the Occupational Pension Scheme and Enterprise Annuity Pension Scheme. The third pillar consists mainly of several individual commercial pension plans. It is a pension plan in which individuals save and invest a portion of their income in various ways. The first pillar has reached a wide coverage level, the second pillar has limited participants, and the third pillar is still in its infancy, with a very low scale and proportion.

Figure 1.1. China Pension Scheme Structure



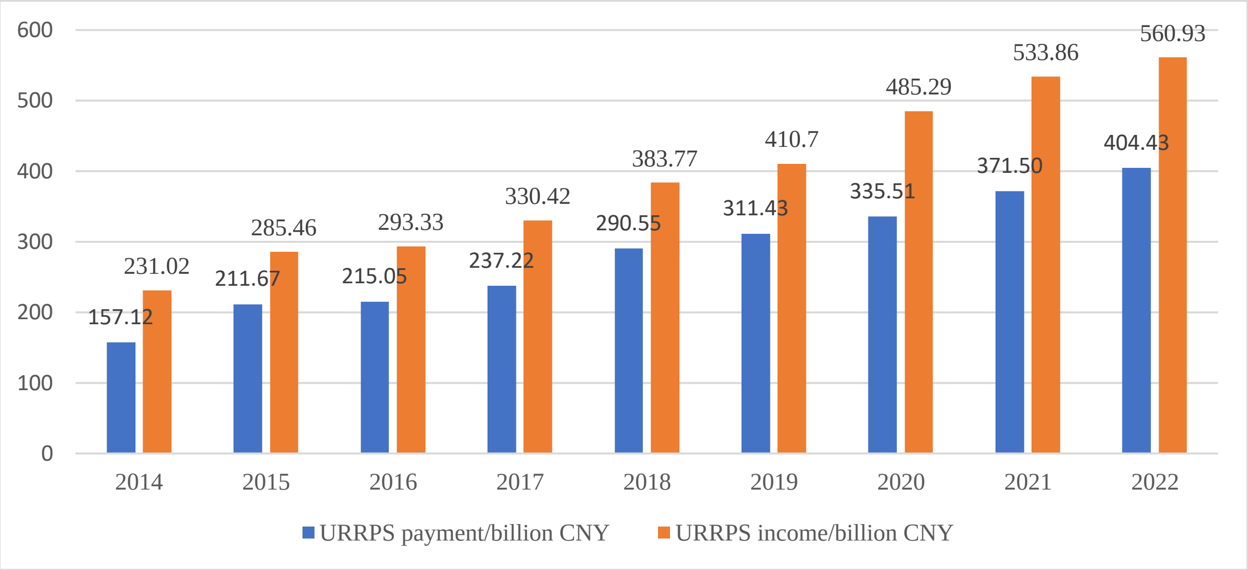
Source: Own Presentation

1.5.1. First Pillar

The first pillar of the pension scheme, often referred to as the basic pension scheme, operates as a pay-as-you-go (PAYG) system and is managed by both central and local governments. China's public pension system has been moving towards achieving universal coverage. Consequently, a series of reforms were introduced between 2009 and 2015. In 2009, the New Rural Social Security (NRSP) was established by China's State Council to provide basic income support for elderly individuals in rural areas, with local offices of the Ministry of Human Resources and Social Security overseeing the funds in individual pension accounts. In 2011, the Urban Residents Pension Scheme (URPS) was created to extend minimum basic income protection to seniors living in urban areas (Cai & Yue, 2020). By 2014, the URPS and NRSP were merged to form the Urban and Rural Residents Pension Scheme (URRPS), further expanding the pension system's reach.

The URRPS, organized by the government, allows for voluntary participation by individuals at least 16 years old (excluding school students) from both urban and rural areas, who are not part of the Urban Employee Pension Scheme, based on their household registration. This scheme employs a financing method that combines individual contributions (ranging from 200 to 2000, varying across regions), collective subsidies, and government subsidies, with local financial support for residents' contributions. The benefits for residents mainly consist of two parts: a basic benefit (DB type) fully subsidized by the government, and an individual account benefit (DC type). The retirement age under the URRPS is 60, and participants are required to contribute for a minimum of 15 years (not necessarily continuous). In 2022, the number of participants reached 549.52 million. The scheme's income and payments from 2014 to 2022 are illustrated in Figure 1.2.

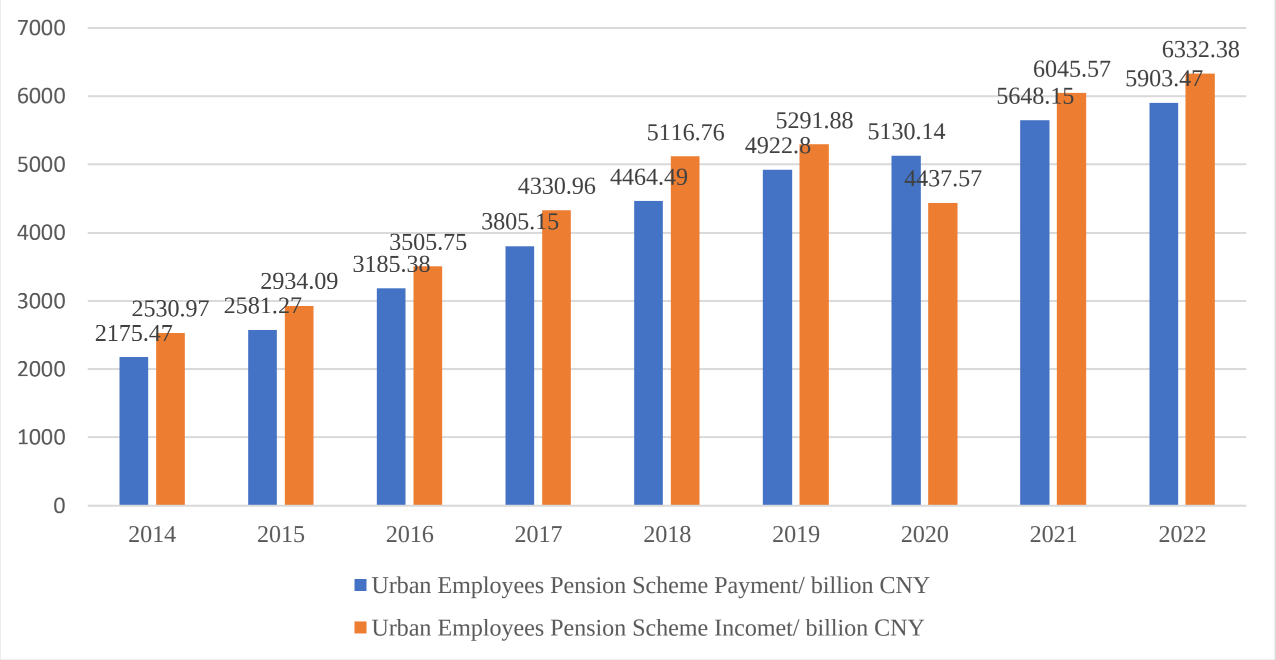
Figure 1.2. URRPS Income and Payment/billion CNY



Data Source: *National Bureau of Statistics* (2024)

The Urban Employee Pension Scheme (UEPS) serves as the most crucial component of the Chinese pension system. This scheme provides insurance benefits either monthly or as a lump sum to employees upon retirement or loss of working ability, taking into consideration their societal contributions and eligibility for pension insurance or retirement conditions. The primary purpose of UEPS is to protect the basic living needs of employees after retirement. In general, employer contributions finance social pensions, while employee contributions are allocated to individual accounts. The contribution rate stands at 20% for employers and 8% for employees. Presently, the scheme operates at the provincial level, with the main participants being urban workers, government employees, and workers in other public agencies. In 2023, the UEPS had 521.21 million participants (*National Bureau of Statistics*, 2024). Figure 1.3 illustrates the income and payments of the Urban Employees Pension Scheme from 2014 to 2022. While the scheme has experienced year-over-year increases in both income and payments, its surplus has faced pressure in recent years. In 2020, the scheme's income even fell below its total payments.

Figure 1.3. Urban Employees' Pension Scheme Income and Payment/billion CNY



Data Source: *National Bureau of Statistics* (2024)

In recent years, the basic pension insurance system has faced growing pressure on its income and payments, while the pension replacement rate remains low. To address these challenges, the country has been working to consolidate and strengthen the foundation of the basic pension insurance system on a large scale. This effort involves implementing various policies, such as providing financial subsidies, transferring a portion of state-owned capital to supplement the social security fund, expanding investment channels for the basic pension fund, and exploring the possibility of delaying the statutory retirement age.

1.5.2. Second Pillar

Occupational pension plans offer supplemental pension benefits to employees in government agencies and other public sectors. These plans are mandatory and designed as a defined contribution (DC) type of pension. Employers contribute 8% of their total salary expenses, while employees contribute 4% of their individual salaries, which is withheld by the employer. Upon retirement, employees can choose how to receive their monthly occupational pension benefits. For those who have settled abroad, the funds in their individual accounts can be paid out as a lump sum. According to MHRSS (2022), there were 72 million participants in enterprise and occupational annuities at the end of 2021.

Enterprise annuities are supplementary pension insurance plans voluntarily established by enterprises and their employees, in addition to their participation in basic pension insurance. These voluntary schemes are defined contribution (DC) type pension plans. There is no set minimum contribution level, but the maximum employer contribution is 12% of wages, while the employee contribution is capped at 8% of wages. As of 2021, 117,500 enterprises in China have established enterprise annuities, with 28.75 million employees participating. At the end of that year, the investment and operation scale of enterprise annuities reached 2.61 trillion yuan, and the annual investment income was 124.2 billion yuan (MHRSS, 2022).

1.5.3. Third Pillar

Currently, China's private pension system and related products are still in their infancy. On April 21, 2022, the programmatic policy for the third pillar of the private pension system, titled "Proposal on Promoting the Development of Private Pensions," was released. It specified that "the state will formulate tax preferential policies to encourage eligible persons to participate in the private pension system and receive private pensions in accordance with the regulations" (Office of the State Council, 2022). On September 26, 2022, the State Council executive meeting further clarified the preferential tax policy for personal pensions, deciding to implement preferential personal income tax for policy-supported, commercially operated personal pensions, with pre-tax deductions for contributors limited to RMB 12,000 per year, no tax on investment income for the time being, and a reduced effective tax burden on income received from 7.5% to 3%. Additionally, the Interim Provisions on Business Management of Individual Pension Investment in Publicly Raised Securities Investment Funds were issued, clearly regulating various market institutions involved in the business of individual pension investment in public funds and their conduct of business promotion.

Table 1.1. Summary of Pension Scheme in China

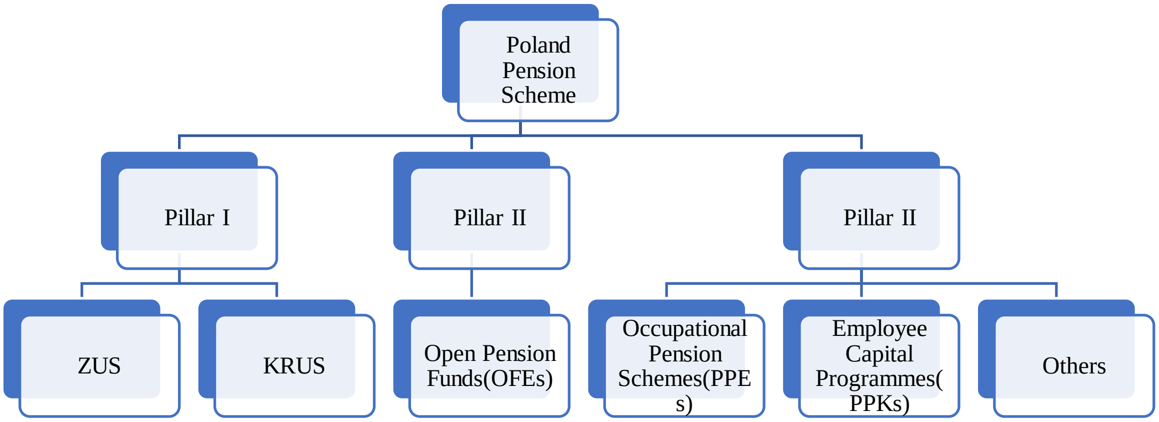
	First pillar		**Second pillar**		**Third Pillar**
	Urban and Rural Residents Pension Scheme (URRPS)	Urban Employees Pension Scheme	Occupational Pension scheme	Enterprise Annuity Scheme	Private
Contribution	The amount of the participant's contribution is added to the local government's subsidy and credited to the individual account. The amount ranges from CNY 100-2000 and will be adjusted according to the local government.	Varies across different regions, the central government recommends employers contribute 20% of wages and employees contribute 8% of the wages.	Contribution is 8% from employers and 4% from employees.	There is no set minimum contribution level, but the maximum is 12% of gross wages and 8% of wages for employers.	Depends on the product type.
Benefit	Basic benefit (DB) individual account (DC type)	Basic benefit (DB) + individual account (DC type)	Individual account (DC) type	DC type	Depends on the product type.
Retirement age	60-year-old	60 years old for men and 50-60 for women, depending on the occupation.	60 for men and 55 for women	60 years old for men and 50-60 for women, depending on the occupation.	Depends on the product type.
Eligibility	Minimum of 15 years contribution	Minimum of 15 serving years		Reach retirement age or lose working ability	Depends on the product type
Coverage	547.97 million participants in 2021	480.75 million participants in 2021 (MHRSS, 2022)	People working in government and other public institutions, 72 million participants	28.75 million participants	
Taxation	Tax free	Tax free	Enjoys Exempt Exempt Taxed (EET) tax treatment, while the employee's tax deduction is capped at 4% of his or her salary during the contribution phase.	Tax deductions are available for 4% (or less) of wages contributed by the employee, and employer contributions are included in the deductions.	EET tax treatment.

Source: Own Presentation

1.6. Facts of Poland Pension Scheme

The current pension system for retirement in Poland is based on a three-pillar arrangement, consisting of a state-run social insurance system and a combination of mandatory and voluntary individual retirement accounts (Kowalewski, 2008). As illustrated in Figure 1.4, the first pillar is the mandatory pillar, primarily managed by the Social Insurance Institution (Zakład Ubezpieczeń Społecznych, ZUS) and the Agricultural Social Insurance Fund (Kasa Rolniczego Ubezpieczenia Społecznego, KRUS), which is mandatory for both non-agricultural and agricultural insured persons. The second pillar, Open Pension Funds (otwarte fundusze emerytalne, OFEs), operated as a mandatory component until the end of June 2014. The third pillar includes Occupational Pension Schemes (pracownicze programy emerytalne, PPEs), Employee Capital Programs (pracownicze programy kapitałowe, PPKs), and other voluntary funded pensions such as Individual Pension Accounts (indywidualne konta emerytalne, IKEs) and Individual Pension Security Accounts (indywidualne konta zabezpieczenia emerytalnego, IKZEs). The purpose of the voluntary third pillar is to complement retirement income.

Figure 1.4. Poland Pension Scheme Structure

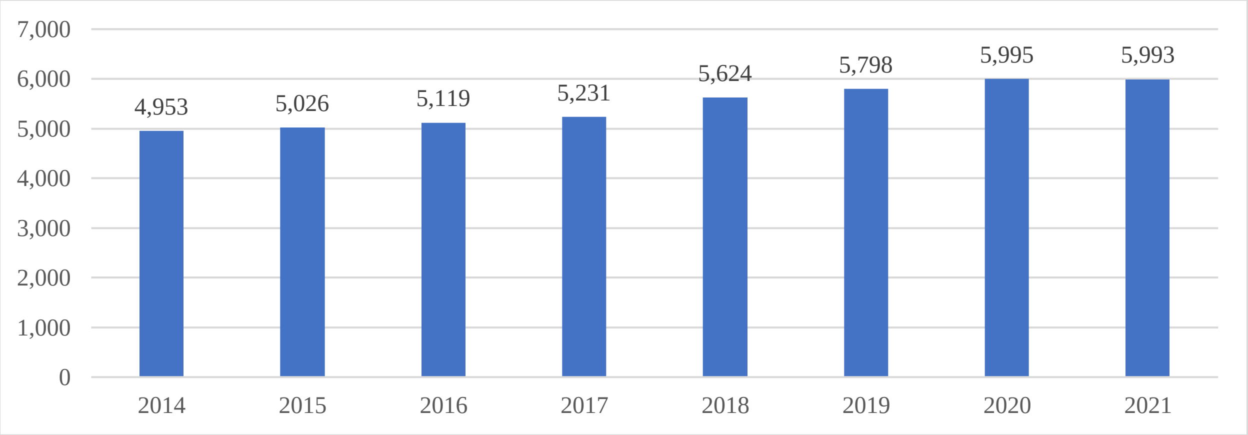


Source: Own Presentation

1.6.1. First Pillar

The first pillar is a pay-as-you-go mandatory scheme, operated by ZUS and KRUS. ZUS manages three funds that are crucial for ensuring the liquidity of social insurance benefits: the Social Insurance Fund (Fundusz Ubezpieczeń Społecznych, FUS), the Bridging Pension Fund, and the Demographic Reserve Fund (ZUS, 2022a). FUS consists of four funds: the old-age pension fund, disability pension fund, sickness fund, and accident fund. In this study, only retirement pension plans for the general public are considered, focusing on the old-age pension fund within the FUS. The normal retirement age is 60 for women and 65 for men. The old-age pension is a DC-type pension with a contribution rate of 19.52% (ZUS, 2022b), divided equally between the payer and the insured person. If the insured person is a member of an Open Pension Fund (OFE) and has submitted a contribution transfer statement, 12.22% of the contribution will be transferred to the FUS account, 4.38% to the FUS sub-account, and 2.92% to the OFE account. The benefit amount is determined by the funds accumulated in the insured person's account. In 2021, the total number of old-age pensioners provided by the FUS was 5,993,000 (see Figure 1.5), representing 15.88% of the population.

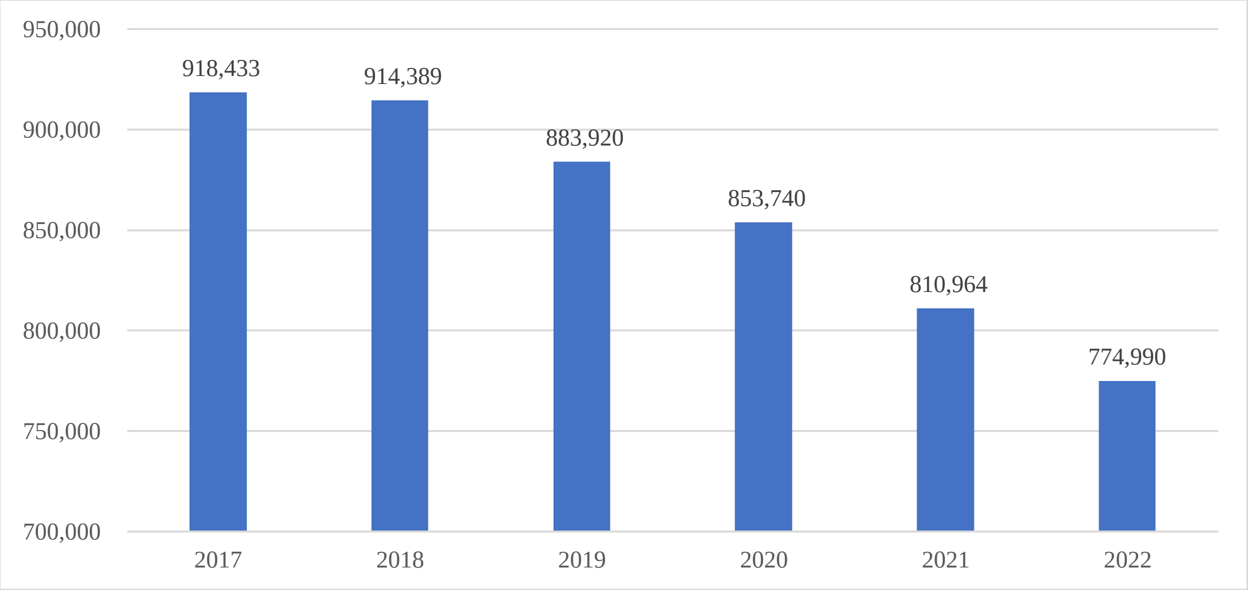
Figure 1.5. Number of old-age pension recipients provided by FUS in 2014-2021 in Thousands



Data Source: ZUS (2022)

In Poland, social insurance services for farmers are managed by an independent institution established by law - the Agricultural Social Insurance Fund (KRUS). Farmer's social insurance comprises two types of mandatory insurance: old-age and disability pension insurance, and accident, sickness, and maternity insurance. These insurances cover the farmer, the farmer's spouse, and farmer's household members. A farmer's old-age pension is granted to insured individuals who have reached the retirement age of 60 for women and 65 for men, provided they have been subject to old-age and disability insurance for at least 25 years. The basic monthly contribution to old-age and disability pension insurance is 10% of the basic pension for the last month of the previous quarter. The farmer's old-age pension benefit consists of a contribution and a supplemental component. The contribution portion is determined assuming 1% of the basic old-age pension (PLN 938.97 from March 1, 2019, to February 29, 2020) for each year of pension insurance. The supplemental part is equal to a proportion (no less than 85%) multiplied by the basic old-age pension. As Figure 1.6 shows, the number of old-age pension recipients in KRUS decreased during the period 2017-2022. In 2022, the total number of old-age pensioners provided by KRUS was 774,990 (see Figure 1.6), or 2.05% of the total population. The total amount of benefits paid by KRUS to old-age pensioners amounted to approximately PLN 14 billion in 2022.

Figure 1.6. Number of old-age pension recipients provided by KRUS in 2017-2022 in Thousands



Data Source: KRUS (2023)

1.6.2. Second Pillar

Up until January 31, 2014, individuals born in 1969 or later were required to enroll in OFEs (Open Pension Funds). However, after that date, the fund became voluntary, allowing people to choose whether to join the funds. Existing OFE members were given the option between April 1 and July

31, 2016, to either remain members of the fund and decide whether to keep their contributions there or transfer them all to ZUS (Social Insurance Institution). Contributions that remain in OFE (Pillar II) will be gradually transferred to ZUS starting ten years before the individual retires. As of the end of 2022, there are 9 OFEs in Poland with about 15 million participants and a total value of approximately PLN 156 billion (UKNF, 2023).

1.6.3. Third Pillar

The third pillar of pensions in Poland had a slow start. Occupational Pension Plans (PPEs) are voluntary pension plans that began operations in 1999. Over the years, the Polish pension system expanded with the addition of new voluntary pension products. PPEs are organized by employers and involve employees for pension purposes. PPE is a DC type pension scheme; under this system, employers must make contributions for their employees, not exceeding 7% of the employee's salary. Additionally, employees may make extra contributions to supplement the employer's contributions, not exceeding 450% of the average monthly wage. All contributions are subject to income tax. The benefit paid to employees after reaching retirement age is determined by the accumulated funds in their account. In 2022, PPEs covered 652 thousand participants with around PLN 19 billion in asset value.

Employee Capital Programs (PPKs) are regulated by the new Act of 4 October 2018 on employee capital plans. This Act, effective from June 2019, establishes a new occupational pension scheme. PPK automatically covers all employees between the ages of 18 and 54. Employees between the ages of 55 and 69 can participate in the PPK program based on a declaration of intent. The PPK will be implemented in four phases, targeting companies of varying sizes, and by the end of 2022, it served around 3 million participants with total annual contributions of PLN 5.33 billion. The accumulated fund value at the end of 2022 was PLN 11.99 billion.

PPK is a DC type pension scheme, with both employees and employers contributing to the employee's account. Employers are required to pay 1.5% of wages, while employees contribute 2.0% of wages to the compulsory part. Voluntary contributions are capped at 2.5% of the employee's salary for employers and 2.0% of the employee's salary for employees. PPK participants also receive subsidies from the Labour Fund. PPK participants can access the accumulated funds in their account after the age of 60.

Other voluntary pension plans, such as IKE and IKZE, are also popular in Poland. Individuals aged 16 and above can contribute to IKE and IKZE. Table 1.2, 1.3, 1.4, and Figure 1.7 show information on pension products available in Poland under Pillar II and Pillar III pension systems from 2015 to 2022, including OFEs, PPE, PPK, IKE, and IKZE. OFEs still dominate all products in terms of the number of participants and fund size. In 2022, OFEs accounted for 75% of the total market in terms of asset value. After the removal of mandatory participation in OFEs in 2014, the number of OFE members declined from 2015 to 2022, with 14.89 million participants in 2022. PPE participation growth slowed after the introduction of PPK in 2019. PPK experienced significant growth after its implementation in 2019 in terms of participant numbers, contribution value, and asset value.

Table 1.2. Number of Participants at the end of 2015-2022(thousand)

Year	**2015**	**2016**	**2017**	**2018**	**2019**	**2020**	**2021**	**2022**
OFEs	16,532	16,424	16,103	15,902	15,670	15,431	15,176	14,891
PPE	393	396	396	426	613	632	641	652
PPK	-	-	-	-	329	1,484	2,548	3,000
IKE	855	903	952	996	951	742	797	800
IKZE	597	643	690	731	654	408	463	476

Data Source: UKNF (2023)

Table 1.3. Value of Contribution at the end of 2015-2022 in PLN million)

Year	**2015**	**2016**	**2017**	**2018**	**2019**	**2020**	**2021**	**2022**
OFEs	3,154	3,203	3,314	3,326	3,470	3,382	3,654	3,959
PPE	1,231	1,210	1,265	1,357	1,896	1,665	1,804	2,809
PPK	-	-	-	-	86	2,205	4,275	5,326
IKE	945	1,066	1,209	1,405	1,703	1,958	2,236	2,367
IKZE	369	474	608	762	929	1,177	1,457	1,446

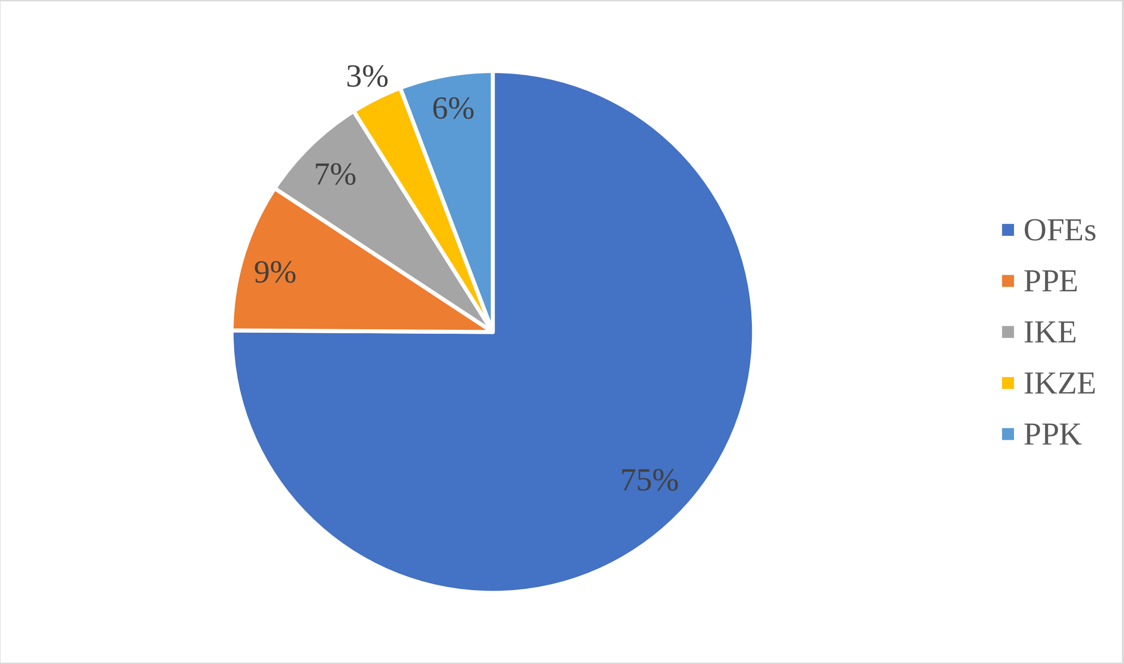
Data Source: UKNF (2023)

Table 1.4. Value of Accumulated Asset at the end of the year 2015-2022 (in PLN million)

Year	**2015**	**2016**	**2017**	**2018**	**2019**	**2020**	**2021**	**2022**
OFEs	140,496	153,434	179,530	157,334	154,816	148,604	187,985	156,378
PPE	10,624	11,395	12,644	12,803	14,548	17,016	18,929	19,129
PPK	-	-	-	-	85	2,818	7,666	11,994
IKE	5,645	6,656	7,961	8,692	10,167	11,924	13,466	14,117
IKZE	620	1,078	1,707	2,318	3,284	4,582	5,977	6,624

Data Source: UKNF (2023)

Figure 1.7. Share of Products by Asset Size in Pillar II and III at the end of 2022



Data Source: UKNF (2023)

Table 1.5. Summary of Pension Scheme in Poland

	First pillar		**Second pillar**	**Third pillar**	
	ZUS old-age pension	KRUS old-age pension	OFE	PPE	PPK
Contribution	The contribution rate is 12.52% and split equally between the payer and the insured person.	10% of the basic pension for the last month of the previous quarter	2.92% of the contribution to FUS can be transfer to OFE account	Employers make up to 7% of the employee's salary +employee voluntarily contribute to 450% of the average monthly wage	Compulsory contribution (employer pays 1.5% of wage + employee pays 2.0% of wage)+voluntary contribution(employer pays up to 2.5% of wage + employee pays up to 2.0%of wage)
Benefit	Determined by the accumulated amount in FUS account, FUS sub-account and OFE account	Contribution part (1% of the basic old-age pension each year and adjusts with different period) + supplemental part (proportion of basic old-age pension)	Determined by the account value	Tax-free at retirement (age 60) in the form of a lump sum or annuity. Annuities. Death benefits are exempt from estate tax.	At retirement (age 60), tax-free if paid over 10 years in: periodic payments, purchase annuity (up to 25% as a lump sum). Otherwise subject to 19% capital gains. Death benefits are exempt from estate tax.
Retirement age	65 for men and 60 for women	65 for men and 60 for women	-	60	60
Eligibility	Minimum contribution of 20 years for women and 25 years for men	Minimum of 25 years contribution	-	At least 25% of staff must participate	Employees who opt out are subject to automated re-enrollment every 4 years
Coverage	5,995,000 recipients in 2020	810,964 recipients in 2021	15.18 million	641 thousand in 2021	2.55 million in 2021
Benefit Taxation	Tax-free	Tax-free	Tax-free	Tax-free	Tax-free if paid over 10 years in scheduled payments, annuity purchase; otherwise, 19% capital gains.

Source: Own Presentation

Chapter 2. Population Dynamics

2.1. Introduction

In the quest to construct an optimal hybrid pension scheme structure for both China and Poland, a comprehensive understanding of the population dynamics is indispensable. This chapter aims to develop a robust population model that takes into account various demographic shifts and trends, which are critical in shaping pension policies and structures. The significance of this model lies in its ability to project future demographic changes, providing a foundational basis for designing pension schemes that are resilient, equitable, and sustainable. The recent demographic trends in China and Poland present unique challenges and opportunities that necessitate a detailed analysis.

One of the cornerstones of the population model is the dramatic improvement in mortality rates witnessed in recent decades. Advances in healthcare, improved living standards, and heightened public health awareness have contributed to increased life expectancy, fundamentally altering the population age structure.

Globally, life expectancy increased from 66.8 years in 2000 to 73.4 years in 2019 (WHO, 2020). However, due to the temporary spike in mortality caused by the COVID-19 pandemic, it dropped to 71 years in 2021. In China, life expectancy continued to rise from 70 in 2000 to 78 in 2021 (The World Bank, 2022a). Conversely, in Poland, although life expectancy increased from 74 to 78 between 2000 and 2019, the pandemic-induced mortality spike resulted in a decrease to 76 by 2021 (The World Bank, 2022b). This shift necessitates a reevaluation of pension schemes, as longer life spans impact the sustainability of pension systems.

China and Poland are witnessing pivotal transitions in their demographic frameworks. Manifested through aging populations and evolving dependency ratios, these shifts are set to significantly impact the design and functionality of pension systems. A meticulously crafted population model is imperative to encapsulate these demographic trends accurately. Such a model is crucial not only for projecting future pension obligations but also for devising adaptable pension plans that can respond effectively to these demographic shifts. Specifically, in China, there has been a notable rise in the dependency ratio from 37% in 2009 to 45% by 2022, signaling an increase in the proportion of dependents relative to the working-age population. Poland's demographic trajectory has followed a similar pattern, with its dependency ratio climbing from 40% in 2010 to a more pronounced 51% in 2022, according to the World Bank (2023a). This intensifies the urgency for pension systems to adjust to an increasing number of retirees relative to the number of active workers.

Both China and Poland are experiencing significant shifts in their population structures. These changes, characterized by aging populations and altered dependency ratios, have profound implications for pension systems. A population model that accurately captures these shifts is essential for forecasting future pension liabilities and designing schemes that are adaptive to these demographic changes. In China, the dependency ratio saw an increase from 37% in 2009 to 45% by 2022. Poland's 40 % in 2010, and increasing to 51% in 2022 (The World Bank, 2023a).

The demographic transformation characterized by an aging population and shifting dependency ratios has amplified the necessity for developing precise mortality models. Such models are instrumental in gauging longevity risk, a key variable influencing the fiscal soundness of pension systems (Pérez-Salamero González et al., 2022). Longevity risk encompasses the uncertainties associated with the lifespans of beneficiaries, directly impacting the pension funds' obligations and stability (Stevens, 2017).

Among the myriad approaches to mortality projection, each is distinguished by unique assumptions and varying degrees of complexity. The ARIMA (AutoRegressive Integrated Moving Average) model, for instance, treats mortality rates holistically, forecasting future rates based on patterns observed in historical data (Costa et al., 2023; Deif et al., 2021; Earnest et al., 2019). It is well-suited for short-term forecasting where age-specific trends are less variable. In contrast, the Lee-Carter model, a seminal approach in demographic forecasting, dissects log mortality rates by age and period, thus facilitating nuanced insights into both temporal dynamics and age-specific mortality trends (R. D. Lee & Carter, 1992; Li et al., 2013; C. Liu & Shi, 2023). It employs Singular Value Decomposition (SVD) to refine parameters, effectively capturing the overarching mortality trends across time as well as the nuanced fluctuations in mortality rates among different age groups.

This chapter opts to employ the Lee-Carter model and its extensions as the primary tools for mortality projection. The Lee-Carter model’s extensions are particularly pertinent; they enhance its robustness in capturing age-specific shifts in mortality improvements over time. By adopting this methodological framework, we aim to deliver a mortality projection that not only mirrors historical declines in mortality but also accommodates the prospective trends that will shape the sustainability of pension systems in the context of China and Poland's evolving demographic landscapes.

The chapter unfolds through a sequence of interconnected sections that collectively offer a panoramic view of the demographic underpinnings essential for pension scheme optimization in China and Poland. Section 2, the Literature Review, delves into the historical and contemporary scholarly works that have shaped our understanding of mortality forecasting. This retrospective provides insight into the evolution of actuarial science and demographic modeling, setting the stage for the subsequent sections. In Section 3, Models, the focus shifts to the analytical frameworks used in mortality projection. The Lee-Carter model, renowned for its demographic forecasting efficacy, is discussed in detail, along with its extensions which introduce geometric and hyperbolic decay factors to reflect time-varying mortality improvements. Section 4, Numerical Results, presents the empirical findings from the application of the mortality models. Finally, Section 5, the Conclusion, encapsulates the insights gained throughout the chapter. It provides a synthesis of the findings from the literature review, model evaluations, and numerical results. This section highlights the implications of the research for pension policy design and offers concluding thoughts on the role of demographic forecasting in the broader context of social security and economic sustainability.

2.2. Literature Review

2.2.1. Mortality Rate Forecasting

In the early twentieth century, the actuarial profession, particularly in England, laid the groundwork for mortality forecasting. Actuaries like Pollard (1987) began assessing the financial implications of increasing life expectancy on pension funds and insurance companies. This era was characterized by deterministic methods, which relied heavily on mathematical formulae and expert judgment. These methods, while foundational, needed to be improved in their ability to adapt to changing demographic trends and underrepresented the inherent uncertainty in future mortality rates.

The 1980s marked the beginning of a paradigm shift towards stochastic methods in mortality forecasting. This shift was significantly influenced by the introduction of the Lee-Carter (LC) method in 1992. Ronald D. Lee and Lawrence R. Carter's paper in the Journal of the American Statistical Association proposed a model that was both parsimonious and powerful in its ability to describe age-time-specific death rates using time series analysis. The LC method was a breakthrough as it accounted for changes over time and age simultaneously, allowing for more dynamic and adaptable mortality forecasts.

The LC model quickly gained prominence due to its simplicity and efficacy. It became a template for numerous adaptations and extensions. For instance, the Booth-Maindonald-Smith (BMS) model, introduced by Booth et al. in 2002, marks a significant advancement. This model refines the analysis of mortality data by adjusting the sample period and uniquely employing a Poisson model to specify kt, the time index crucial in mortality forecasts. This innovative approach allows the BMS model to more accurately reflect contemporary and anticipated mortality trends, accommodating rapid changes in demographics and public health. It stands in contrast to models like the Hyndman-Ullah (HU) model, proposed by Hyndman and Ullah in 2007, which integrates a broader array of functional principal components to decompose mortality improvements. The HU model's emphasis on a more detailed component analysis aids in uncovering intricate patterns and influences within mortality data.

The success of the LC model paved the way for a more diverse range of stochastic models. Researchers began exploring various statistical and computational techniques to capture the complexities of mortality trends. For instance, Booth and Tickle (2008) discussed the evolution of these methods, emphasizing the growing importance of stochastic models in capturing uncertainty and variability in mortality forecasts. This period also saw the integration of Bayesian statistical methods, as noted by Raftery et al. (2013), which offered a framework for incorporating prior knowledge and probabilistic reasoning into mortality predictions.

Understanding cohort effects has become increasingly important in mortality forecasting. Bell's (1997) research highlighted the significance of cohort analysis in capturing the peculiarities of different generational experiences. These effects, which encompass variations in mortality risks experienced by individuals born in the same period, have been shown to significantly impact mortality trends (Yang, Fu, & Land, 2004). Recent studies have focused on integrating cohort analysis into existing models for more nuanced predictions. Li and Lee (2005) expanded on this by examining the impact of historical events and lifestyle changes on different cohorts. Further, Murphy (2010) explored the long-term implications of cohort effects on life expectancy, providing insights into the complex interplay of historical, social, and biological factors.

More recently, Li et al. (2013) address the phenomenon of "rotation" in age patterns of mortality decline, proposing an extension to better project long-term mortality trends, especially in populations with high life expectancies. They demonstrate the utility of this approach with data from Japan and the United States, suggesting it could lead to more accurate age-specific mortality projections. The article by Hong et al. (2021) introduces a novel approach to forecasting mortality rates by combining the Lee-Carter model with machine learning techniques, specifically Random Forest (RF) and Artificial Neural Networks (ANN). Focusing on Malaysian mortality data from 2000 to 2016, the study reveals that hybrid models, LC-ANN for males and LC-ARIMA for females, outperform traditional models in accuracy. Furthermore, the LC-ARIMA model is highly effective in countries with advanced healthcare and longer life expectancies, while LC-ANN excels in regions with less efficient healthcare systems. Liu and Shi (2023) build on this by introducing the LC-V model, designed to overcome the lack of age coherence in existing forecasts. Through two innovative frameworks and three extensions, they ensure dynamic and age-coherent forecasting, with empirical results from 15 countries showcasing superior accuracy against existing models, including in scenarios involving multiple populations.

2.2.2. Application of Mortality Model in Pension Scheme

Mortality forecasting significantly impacts policy-making, especially in the realm of pension planning and social security systems. Its applications within various governance frameworks are pivotal for policy development and execution. Boado-Penas et al. (2008) and Vidal-Meliá and Del Carmen Boado-Penas (2013) have demonstrated how mortality improvements necessitate adjustments in European pension schemes, revealing the diverse policy responses and their effectiveness. They further explore the Contribution Asset (CA) and Hidden Asset (HA) within Pay-As-You-Go (PAYG) systems, evaluating their reliability as solvency indicators in the Actuarial Balance sheet, considering factors like inflation and mortality rates.

Yao et al. (2016) extend this discussion by optimizing investment strategies for defined contribution pension funds amidst market variability and mortality risk. Employing a Markovian regime-switching model, the study underscores the need for dynamic investment strategies responsive to economic cycles and demographic shifts. Baltas et al. (2022) focus on managing pension funds during the distribution phase, emphasizing the challenges posed by inflation, mortality, and model uncertainty, and offer robust control and dynamic programming solutions for optimal investment decisions.

Shi and Kolk (2022) investigate the interplay between mortality and pension inequality, utilizing Swedish data to analyze the effects of education and pre-retirement earnings on lifetime pensions. Their findings highlight the significant role mortality differences play in exacerbating pension inequality, emphasizing the need for policies that consider both pension structure and demographic factors. Collectively, these studies illuminate the complex relationship between mortality forecasting and pension system sustainability, providing valuable insights for policy formulation and financial management within social security frameworks.

2.3. Models

2.3.1. Lee-Carter model

The Lee-Carter (LC) model, initially proposed by Lee and Carter (1992), has been widely recognized as a seminal approach for modeling and forecasting mortality rates. Its core principle lies in decomposing log mortality rates into age and time components, allowing for an analysis of temporal trends and age-specific patterns in mortality. The model is mathematically represented as follows:

$$\ln m_{x,t} = a_x + b_x \cdot k_t + \varepsilon_{x,t} \tag{2.1}$$

Where:

- $\ln\ m_{x,t}$ denotes the natural logarithm of the mortality rate at age x in year t.
- a_x is the age-specific intercept, representing the average log mortality rate over time for each age group.
- b_x indicates the sensitivity of the log mortality rate at age x to changes in the time index.
- k_t is the time index, capturing the general level of mortality at time t.
- $\epsilon_{x,t}$ is the error term, accounting for random variations and model deviations.

The LC model estimates its parameters using Singular Value Decomposition (SVD). This non-linear model treats age and time as categorical variables, assuming a homoscedastic and normally distributed error term. Traditional regression methods are unsuitable for the LC model due to the absence of known covariates; it relies on unknown parameters. Lee and Carter applied SVD to derive an Ordinary Least Squares (OLS) solution. They first centered the matrix by subtracting the age-specific average logarithm of the death rate from each column of the natural logarithm of the death rates. Then, using the first term of the SVD, they determined b_x (the age-specific pattern of mortality change) and k_t from the singular vectors and values.

For h-step-ahead forecasting within the LC model where k_{T+h} is modeled as a random walk with drift, the forecasted log mortality rate at age x for h steps ahead of the last observed year T is given by:

$$\ln m_{x,T+h} = a_x + b_x \cdot (k_T + h \cdot \text{drift}) \tag{2.2}$$

where:

- a_x is the fixed age-specific average log mortality rate over time.
- b_x is the fixed sensitivity of the log mortality rate at age x to changes in the time index.
- k_T is the last observed value of the time index.
- drift is the estimated average change in k_t per time step, which is constant and represents the trend in the time index.
- h is the number of time steps into the future for the forecast.

2.3.2. Extensions of the LC Model

In the LC model, the b_x coefficients representing age-specific mortality improvement rates are traditionally considered constant. However, R. Lee and Miller (2001) and Rau et al. (2008) observed that these coefficients might vary over time, noting shifts in mortality trends throughout the 20th century: early on, young ages saw significant improvements, while later, older ages experienced greater reductions in mortality. This variation presents a challenge for the LC model, particularly in long-term forecasting. To address this, Li et al. (2013) proposed a "rotation" in the b_x schedule within the LC model, allowing it to better reflect these changing mortality patterns over time.

The Lee-Carter with Geometric Adjustments (LC-G) model, an extension of the LC framework proposed by Gao and Shi (2021), introduces a geometric decay factor into the b_x coefficients. This addition enables the model to more accurately reflect time-varying mortality improvements across different age groups. Utilizing an autoregressive (AR) framework, specifically AR(1), the LC-G model allows the b_x coefficients to geometrically converge to a long-run mean. The time-varying $\hat{b}_{x,h}^{G}$ is formulated as:

$$\hat{b}_{x,h}^{G} = r_x^h\left(\hat{b}_x - \hat{B}\right) + \hat{B} \tag{2.3}$$

where $\hat{B}$ is the long-run mean, and r_x is the age-specific geometric decay factor, constrained between 0 and 1 for stationarity. The model sets $\hat{b}_{x,0}^{G} = \hat{b}_x$ and adjusts $\hat{b}_{x,h}^{G}$ over time, ensuring $\hat{b}_{x,h}^{G}$ converges to $\hat{B}$ geometrically. The convergence speed varies inversely with r_x : larger values slow it down, while smaller values speed it up.

In time-series analysis, hyperbolic decay is associated with the concept of long memory, characterized by a decay speed slower than short memory processes like geometric or exponential decay. Feng et al. (2021) applied this concept to mortality modeling and forecasting. In their approach, the time-varying $\hat{b}_{x,h}^{H}$, under hyperbolic decay, is defined as:

$$\hat{b}_{x,h}^{H} = \lambda_h(\delta_x)\left(\hat{b}_x - \hat{B}\right) + \hat{B} \tag{2.4}$$

Here, $\lambda_k(\delta_x)$ is a recursive function given by:

$$\lambda_k(\delta_x) = \frac{k - 1 + \delta_x}{k}\lambda_{k-1}(\delta_x) \text{ with } \lambda_0(\delta_x) = 1 \tag{2.5}$$

The hyperbolic parameter δ_x, ranging between 0 and 1, dictates the decay rate. As h approaches infinity, $\lambda_h(\delta_x)$ trends towards zero, ensuring that $\hat{b}_{x,h}^{H}$ eventually converges to the long-run mean $\hat{B}$. This leads to age-coherent forecasts, with the speed of decay being inversely related to δ_x : a larger δ_x results in slower decay, while a smaller δ_x leads to a faster convergence to $\hat{B}$.

Estimating r_x and δ_x for the LC-G and LC-H models is challenging due to their agedependent nature and the absence of in-sample data for fitting. Followed by Gao and Shi (2021), the super-smoother method is used to smooth $\hat{b}_x$ values from the LC model, employing the inversed Epanechnikov kernel, $1 - K_\beta(\tau - 1)$, where $\tau = x/N$ and β is the bandwidth parameter. This results in r_x and δ_x being modelled as:

$$\begin{aligned} r_x &= r_1 \left(1 - K_\beta(\tau - 1)\right) \\ \delta_x &= \delta_1 \left(1 - K_\beta(\tau - 1)\right) \end{aligned} \tag{2.6}$$

For $\beta < 1, r_x$ and δ_x start equal to r_1 and δ_1 and then increase with age x, ensuring a declining speed of convergence and capturing different mortality improvement rates across ages (Liu & Shi, 2023).

In this study, LC-G and LC-H, are thoroughly evaluated to identify optimal tuning parameters r_1 and δ_1, as well as the ideal bandwidth for the inversed Epanechnikov kernel. This is achieved through a grid search method aimed at minimizing the Root Mean Square Forecast Error (RMSFE) within the in-sample period from 1997 to T. Following the methodology proposed by Liu and Shi (2023), RMSFE, particularly for h-step-ahead forecasting, is calculated using the following formula:

$$\text{RMSFE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n} (\hat{y}_{t+h} - y_{t+h})^2}$$

Here, $\hat{y}_{t+h}$ represents the forecasted value for a time point $t + h$ ahead, y_{t+h} is the actual value at that future time point, and n denotes the number of observations in the testing set.

To determine the most accurate forecasting model among the LC, LC-G, and LC-H models, the dataset is partitioned into a training set (1997 to 2017) and a testing set (2018 to 2022). The performance of each model is assessed based on the lowest RMSFE observed in the testing set, focusing on h-step-ahead forecasts. The model that exhibits the best predictive capability, as indicated by the minimum RMSFE, will then be employed for simulating mortality rates for future years.

The methodologies described in this chapter are implemented using Python. Partial code snippets covering aspects like data handling, model application, and forecasting are included in Appendixes.

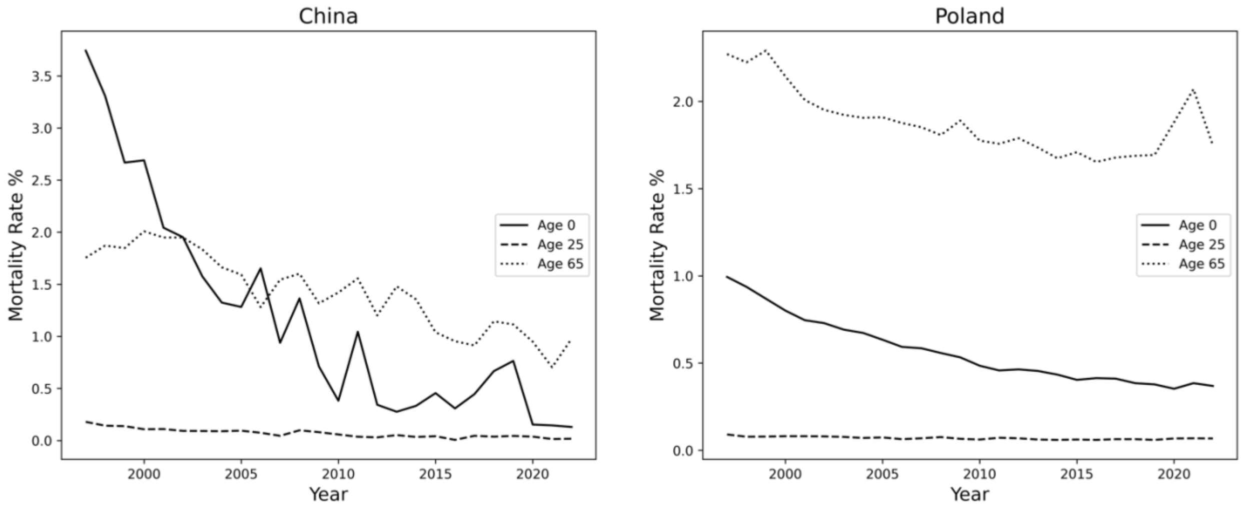
2.4. Numerical Results

2.4.1. Data

Regarding China, the study utilizes mortality data from the China Population and Employment Statistics Yearbook, as published by the National Bureau of Statistics (2023). Notably, the scope of the data for China is confined to mainland China, deliberately excluding the Hong Kong, Macao Special Administrative Region, and Taiwan Province. For Poland, the mortality data is extracted from the Human Mortality Database (2023), aligning with the methodology adopted by Sharrow and Anderson (2016).

The study utilizes mortality rate data from 1997 to 2022 for ages 0 to 89 and 90+, with this period chosen due to standardized age categorization. Prior to 1997, mortality data was categorized differently, with the age groups ending at 84 and a combined category for those aged 85 and older. Figure 2.1 displays mortality trends for ages 0, 25, and 65, illustrating demographic shifts over time. For both China and Poland, the mortality rates for ages 0, 25, and 65 generally display a declining trend over the study period. However, in 2021, an anomaly is observed due to the impact of the COVID-19 pandemic. Specifically, Poland experienced an overall increase in mortality rates across these age groups, reflecting the pandemic's significant impact. Conversely, China's mortality rates continued to decline in 2021, indicating a differing impact or response to the pandemic in this region.

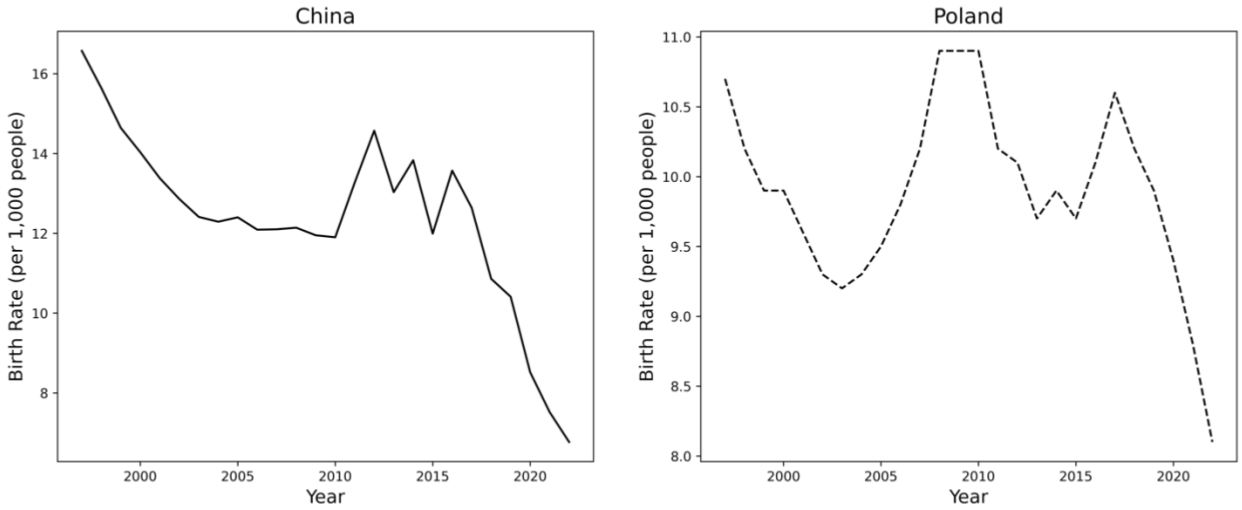
Figure 2.1. Mortality Rate (%) for China and Poland over 1997 to 2022



Data Source: National Bureau of Statistics (2023) and Human Mortality Database (2023)

Figure 2.2 delineates the birth rate (per 1,000 people) trends for China over the span of 1997 to 2022, showcasing distinct shifts in demographic patterns. From 1997 to 2011, the birth rate in China gradually falls from 16.57 to 13.27, indicating a period of steady demographic change. An interesting deviation is observed in 2012, where the birth rate momentarily rises to 14.57, before resuming its decline. Subsequently, a notable acceleration in the reduction of birth rates is evident, leading to a sharp decrease to 6.77 by 2022, highlighting a significant demographic transition within the country. Conversely, Poland's birth rate demonstrates greater consistency, starting at 10.7 in 1997. It fluctuates modestly, achieving relative stability between 2007 and 2010 with rates around 10.9, and then it gradually diminishes to 8.1 by 2022, reflecting a different pattern of demographic evolution compared to China.

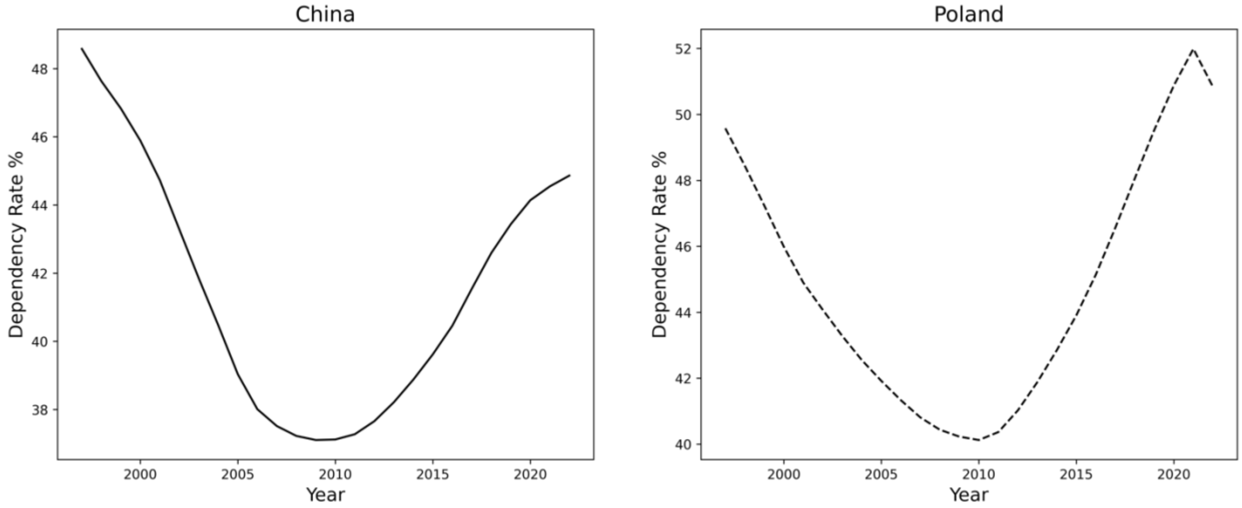
Figure 2.2. Birth Rate (per 1,000 people) for China and Poland over 1997 to 2022



Data Source: National Bureau of Statistics (2023), Statistics Poland (2024) and The World Bank (2023b)

Figure 2.3 depicts the age dependency ratios (%) for China and Poland from 1997 to 2022, highlighting the dynamic balance between the dependent population and those of working age. The age dependency ratio is calculated in accordance with the World Bank's (2023) definition, which considers the proportion of dependents—individuals aged below 15 and above 64—to the working-age population, defined as those between the ages of 15 and 64. In China, the dependency ratio saw a decline from 48.58% in 1997 to a low of 37.10% in 2009, before ascending to 44.86% by 2022. Poland's trajectory was similar, with its ratio decreasing from 49.58% in 1997 to 40.12% in 2010, and subsequently increasing to 50.84% in 2022. These trends indicate that post-2010, both countries have experienced a growing demographic pressure, with a relatively smaller working-age population bearing the increasing responsibility of supporting both the younger and older dependents. This shift suggests emerging challenges in societal support structures and economic sustainability as the proportion of dependents rises.

Figure 2.3. Age Dependency (%) for China and Poland over 1997 to 2022



Data Source: The World Bank (2023a)

2.4.2. Model Selection for Mortality Rate Projection

In this study, the mortality data is split into a training set spanning from 1997 to 2017 and a testing set from 2018 to 2022. This division allows for the evaluation and comparison of the forecasting performances of three different models: the Lee-Carter (LC), Lee-Carter with Geometric decay (LC-G), and Lee-Carter with Hyperbolic decay (LC-H). The RMSFE is used as the metric for assessing the accuracy of these models. Lower RMSFE values indicate more precise forecasts. Table 2.1 reveals that for both China and Poland, the LC-H model consistently demonstrates superior forecasting accuracy with the lowest RMSFE values (3.1181 for China and 3.6527 for Poland). In contrast, the standard LC model shows the highest RMSFE values (3.5698 for China and 3.9700 for Poland), suggesting less accurate predictions. The LC-G model falls between these two in terms of accuracy. These findings imply that incorporating hyperbolic decay into the Lee-Carter model significantly enhances its ability to predict mortality rates, making the LC-H model the preferred choice for forecasting future mortality rates in both countries.

Table 2.1. RMSFE of Three Models for the Testing Sample in China and Poland

Models	China	Poland
LC	3.5777	3.9380
LC-G	3.1516	3.6345

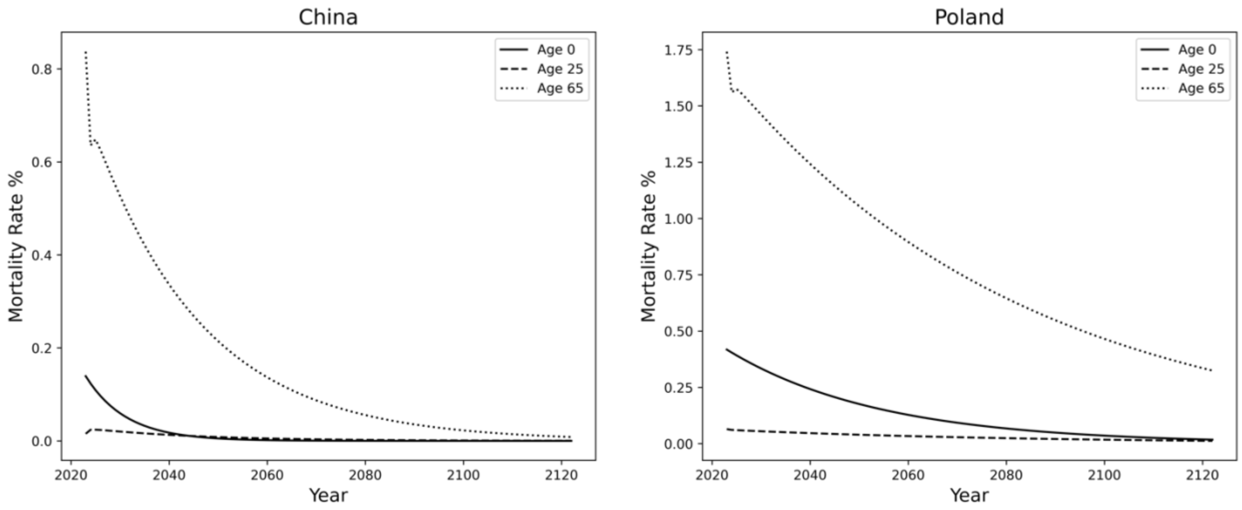
LC-H	3.1181	3.5973

Data Source: Own Presentation

2.4.3. Future Mortality Rate Projection

For the projection of future mortality rates, the LC-H is selected based on its demonstrated forecasting accuracy over other models. This model is trained using the complete data set from 1997 to 2022, encompassing both China and Poland. The focus is on simulating mortality rates for all ages up to 89, as well as the age group 90+, projecting these rates 65 years into the future, from 2023 to 2122. The out-of-sample forecasting conducted by the LC-H model extends beyond the latest available data. Figure 2.4 provides a detailed projection of mortality rates (%) for China and Poland, spanning from 2023 to 2122, across three pivotal ages: 0, 25, and 65. This bifurcated display, with China on the left and Poland on the right, facilitates a direct comparison of future demographic and health trajectories between these two countries. The data demonstrate a clear trend of decreasing mortality rates across the century for both nations, albeit with distinct patterns for different age groups. For newborns (age 0), the forecast shows an initial low mortality rate that declines further over time, with China's rates decreasing more sharply than those of Poland. Mortality rates for the 25-year-old cohort are depicted as consistently low and stable, highlighting an enduring low mortality risk among young adults in both countries. Conversely, the forecast for individuals aged 65 reveals greater fluctuation, indicative of the varying health challenges that accompany older age. This age group's mortality rates are initially higher but are projected to decrease, pointing to an overall improvement in longevity and health outcomes for the elderly as the century progresses.

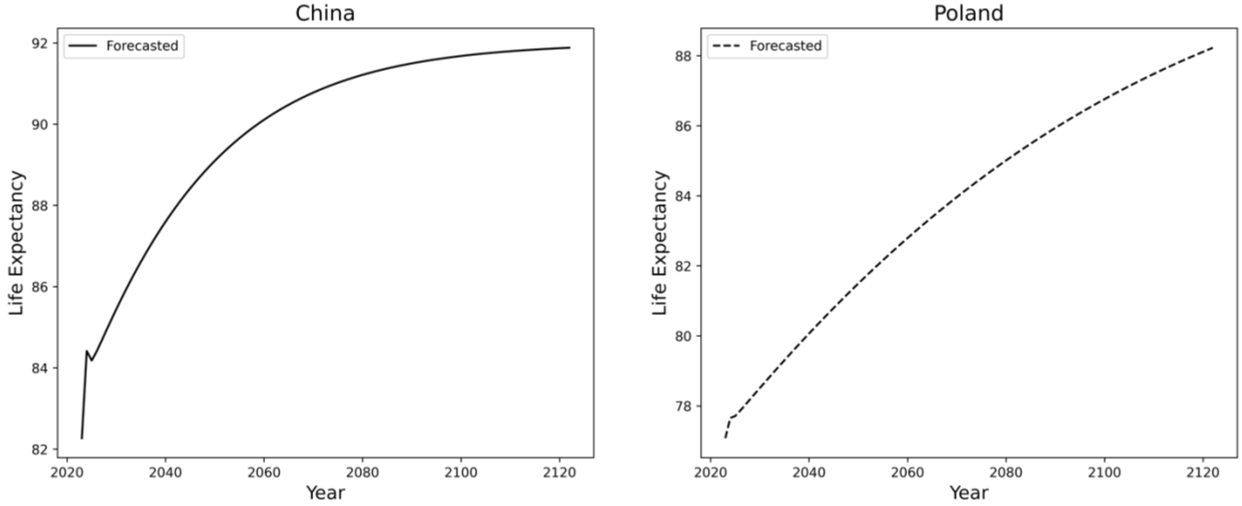
Figure 2.4. Forecasted Mortality Rate (%) for China and Poland over 2023 to 2122



Data Source: Own Presentation

Figure 2.5 displays the forecasted life expectancy trends for China and Poland over a period extending from 2023 to 2122. The data reveals a continuous upward trend in life expectancy for both countries, with China's life expectancy consistently higher than Poland's throughout the forecast period. Starting in 2023, China's life expectancy is approximately 82.27 years, gradually increasing over the century. Poland's life expectancy begins at about 77.08 years in 2023, also showing a steady increase but remaining lower than China's across the timeframe. The fluctuations in forecasted life expectancy between 2023 and 2025 are attributed to the anomalous actual mortality rates observed during the COVID-19 pandemic. This period's elevated mortality rates have influenced the projection models, resulting in adjustments to the life expectancy estimates for these years.

Figure 2.5. Forecasted Life expectancy for China and Poland over 2023 to 2122



Data Source: Own Presentation

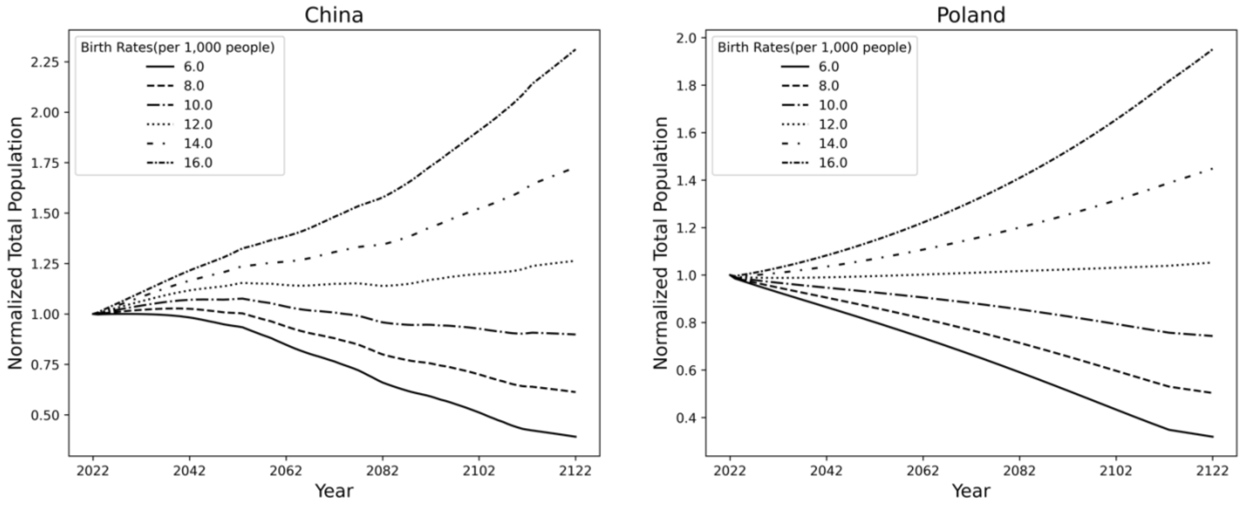
2.4.4. Population Structure

Utilizing birth and forecasted mortality rate data, this section aims to project the future population structure for each individual age. The analysis is meticulously structured to provide a detailed examination of demographic shifts over the future century, assessing how varying birth rate scenarios may influence the age distribution within the populations of China and Poland. This section delves into the composition and trends across different age groups, with a particular emphasis on the ratios of dependents—individuals younger than 15 or older than 64—to the working-age demographic, those aged 15-64. Different birth rate assumptions—low, moderate, and high—are explored to evaluate their long-term impact on population structures. By projecting and examining the future population structure, this section aims to lay a demographic foundation for understanding the long-term fiscal viability and the strategic design of pension plans that equitably balance and distribute risks and rewards across generations.

As shown in Figure 2.2, the historical birth rates for China and Poland from 1997 to 2022, which ranged from 6.0 to 17.0 births per 1,000 people. Based on this historical range, Figure 2.6 sets a spectrum of birth rates from 6.0 to 16.0 per 1,000 people to model how variations in birth rates could influence population changes. The base year of 2022 serves as the normalization point, with its population indexed to 1 for comparative purposes. The left subplot in Figure 2.6 illustrates the population trajectories for China under different birth rate assumptions. With a birth rate of 6.0

per 1,000 people, closely mirroring the actual rate in 2022, the population is projected to decline until 2025, followed by a slight uptick until 2029, and then a continued decline thereafter. At a birth rate of 8 per 1,000, the population is expected to increase until 2039 before it begins to diminish. With a rate of 10 per 1,000, the increase continues until 2053, after which a decline is projected. For a birth rate of 12, the population growth is sustained until 2058 before it starts to decrease; meanwhile, birth rates of 14 and 16 per 1,000 consistently lead to population increases throughout the period. The right subplot displays Poland's population changes using the same range of birth rates. Here, birth rates of 6.0, 8.0, and 10.0 per 1,000 are projected to result in a population decline over the next century, with the 8.0 rate approximating the actual birth rate in 2022. For a birth rate of 12.0 per 1,000, the population is expected to decrease until 2032 and then rise in the subsequent years. With a rate of 14 births per 1,000, a population decrease is anticipated until 2024, followed by an increase, and for a rate of 16 births per 1,000, the population is forecasted to decrease only in 2023, then experience growth thereafter.

Figure 2.6. Normalized Population from 2022 to 2122 for China and Poland



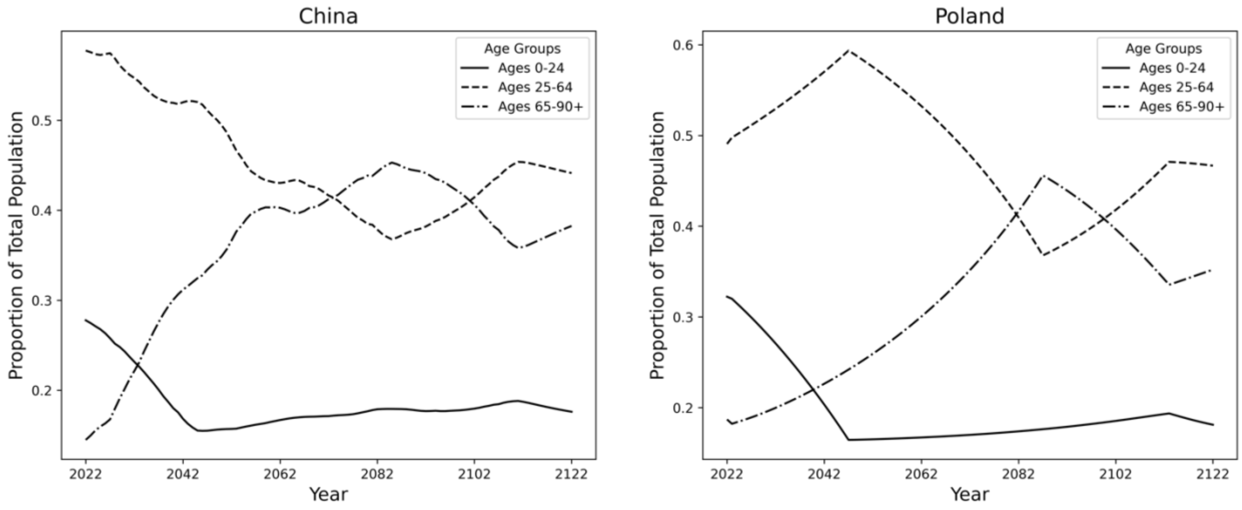
Data Source: Own Presentation

Note: For comparative analysis, the population size in the base year of 2022 is normalized to a value of 1.

Figure 2.7 offers a detailed comparative analysis of the proportional age distributions from 2022 to 2122 for China and Poland, predicated on a low birth rate scenario of 6 births per 1,000 people. For China, the youngest age cohort's proportion (0-24) initially shows a precipitous decline due to the low birth rate. This trend is later partially offset by a minor uptick, potentially reflecting a decrease in mortality rates. The proportion of the working-age group (25-64) exhibits variability, with a prevailing downward trend that continues until around 2082, after which it shifts to an upward trend persisting until about 2112, and then it declines once more. The oldest age group's proportion (65-90+) in China follows a similar pattern, with an increase until around 2082, a decrease leading up to 2112, and a subsequent increase.

In contrast, the trends for the 0-24 and 65-90+ age group proportions in Poland are more pronounced than in China, indicating a sharper impact of the low birth rate on Poland's population structure. The fluctuations in these age groups are more marked, signaling clear demographic transitions. However, for the 25-64 age group in Poland, the proportion increases from 2022 until around 2042, suggesting a growing working-age population initially. This is followed by a decline until around 2082 and then a rise again until approximately 2112 before decreasing.

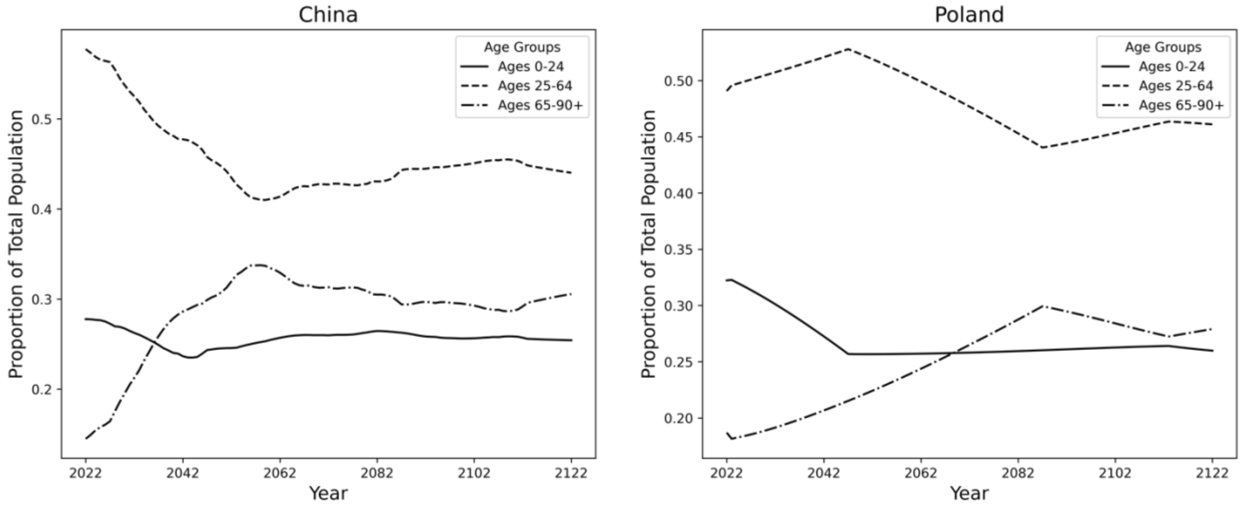
Figure 2.7. Low Birth Rate Scenario-Age Distribution from 2022 to 2122 for China and Poland



Data Source: Own Presentation

Figure 2.8 illustrates the age group proportions in China and Poland under a moderate birth rate scenario of 10.0 births per 1,000 people. Compared to the low birth rate scenario, the patterns here are more stable without the previous intersections between the 25-64 and 65-90+ age groups. However, intersections are still present between the youngest (0-24) and the oldest (65-90+) age cohorts for both countries, indicating a point where the proportions of the youngest and oldest are equivalent.

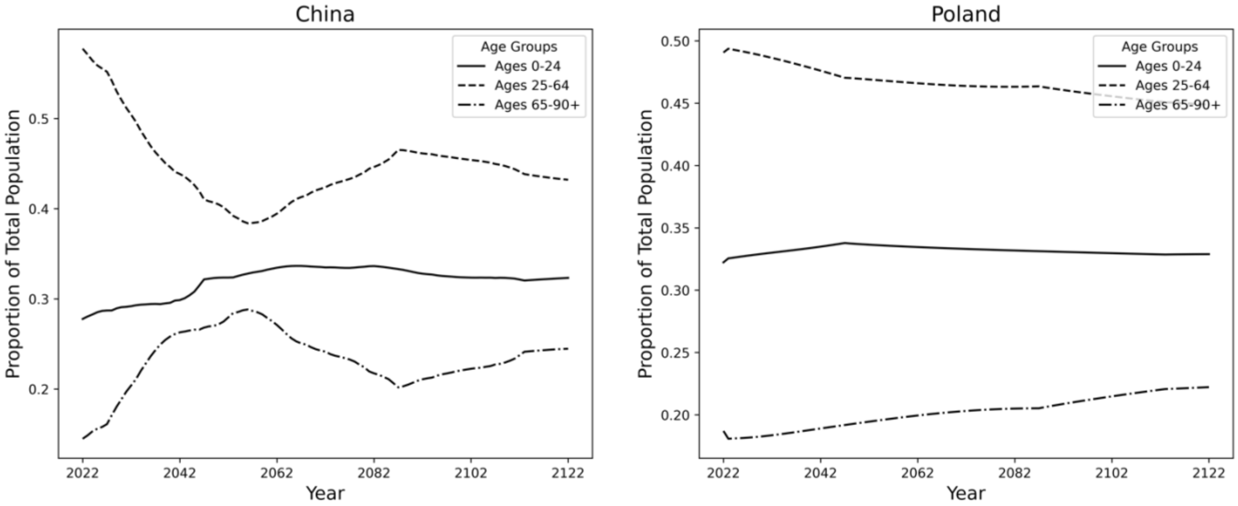
Figure 2.8. Moderate Birth Rate Scenario-Age Distribution from 2022 to 2122 for China and Poland



Data Source: Own Presentation

Figure 2.9 presents the proportions of different age groups under a high birth rate of 14.0 births per 1,000 people. In this scenario, the age groups do not intersect in either country, signaling distinct and separate trajectories for each cohort. For China, the working-age group (25-64) sees a decline until around 2062, followed by an increase until approximately 2087, and then another decline. The youngest age group (0-24) notably increases in proportion until around 2047, after which it stabilizes around a proportion of 0.35. The oldest age group (65-90+) experiences an increase in proportion until around 2052, then a decrease until 2087, before rising again. In Poland, the working-age group's (25-64) overall trend is a gradual decline, moving from 0.50 to 0.45, not as abrupt as in other scenarios. The youngest age group (0-24) slightly increases until around 2057, followed by a minor decrease, maintaining an overall proportion around 0.32. For the elderly age group (65-90+), the proportion generally continues to increase. Overall, for both countries, the 25-64 age group is consistently projected to have the highest proportion over the next 100 years, the 0-24 age group has the second highest proportion, and 65-90+ is consistently the lowest.

Figure 2.9. High Birth Rate Scenario-Age Distribution from 2022 to 2122 for China and Poland



Data Source: Own Presentation

2.5. Conclusion

This chapter presents a comprehensive analysis of population dynamics to guide the construction of an optimal hybrid pension scheme for China and Poland. Central to this endeavor has been the development of a robust population model that integrates key demographic shifts and trends, essential for informed pension policy and structural design. A significant aspect of our study is the examination of improved mortality rates, influenced by advances in healthcare, elevated living standards, and enhanced public health awareness. These developments have led to increased life expectancy, thereby altering the population's age structure. This shift necessitates a reassessment of pension systems to maintain their sustainability in light of extended lifespans.

In the realm of demographic modeling, particularly for pension scheme design, the introduction of the Lee-Carter with Geometric decay (LC-G) and Lee-Carter with Hyperbolic decay (LC-H) models marks a significant advancement, especially when considering the rotation of the b_x coefficients. Traditional mortality models often struggle to account for the evolving nature of mortality improvement rates across different age groups over time. The LC-G model addresses this by incorporating a geometric decay factor into the b_x coefficients, enabling the model to capture more nuanced, time-varying mortality trends. More importantly, the LC-H model, through its hyperbolic decay approach, offers an even more refined mechanism for modeling these trends. This approach allows for a more realistic rotation of the b_x schedule, reflecting the historical shifts where younger ages initially saw more significant improvements, while more recently, older age groups have experienced greater reductions in mortality. This rotation in the b_x coefficients within the LC-H model is crucial for accurately forecasting long-term mortality rates and, by extension, for developing pension schemes that are responsive to the dynamically changing demographic structures, particularly in the context of aging populations and shifting life expectancies. Through empirical analysis using data from China and Poland from 1997 to 2022, our findings reveal the superior predictive capability of the LC-H model. This is evidenced by its lower Root Mean Square Forecast Error (RMSFE) values, making it a more reliable tool for projecting future pension liabilities and designing adaptable pension schemes.

The in-depth examination of birth rates within this chapter, particularly through the lens of the numerical results section, highlights their significant implications for the future population structures of China and Poland. Our analysis, projecting over a century, demonstrates how different birth rate scenarios—ranging from low to high—could distinctly sculpt the demographic landscape of these nations. For instance, the projections indicate that with a birth rate of 6.0 per 1,000 people, closely mirroring China's rate in 2022, the population is anticipated to decline, underscoring the need for pension systems to adjust to a shrinking base of contributors. Conversely, a higher birth rate scenario of 14.0 per 1,000 people suggests a potential for population growth, altering the dependency ratio dynamics and the financial sustainability of pension schemes.

These findings are pivotal for pension planning, underscoring the necessity of embedding dynamic demographic parameters into the actuarial models that underpin pension systems. By doing so, policymakers can better anticipate the future demographic shifts that will impact the balance between the working-age population and dependents, a critical factor in the long-term viability of pension funds. This nuanced understanding of demographic trends, informed by specific numerical results, is essential for crafting pension schemes that are financially sound and socially sustainable, capable of adapting to the evolving demographic realities of China and Poland.

In synthesis, this chapter has not only provided a detailed exposition of demographic trends and their implications for pension systems but has also highlighted the critical role of sophisticated mortality and population models in forecasting and planning. As we move forward, the insights gleaned here will undoubtedly serve as a cornerstone for the development of pension schemes that are not only attuned to current demographic realities but are also forward-looking, capable of adapting to future demographic shifts. The imperative for pension systems to be flexible, equitable, and sustainable in the face of demographic changes has never been more pronounced, underscoring the need for continued research, innovation, and policy adaptation in this vital area of economic and social policy.

Chapter 3. Hybrid Pension Scheme Potential for Risk Reduction

3.1. Introduction

The ongoing increase in life expectancies and global population aging has placed significant stress on global pension systems. This stress has resulted in pressing concerns about the long-term sustainability of these systems, as well as the equitable distribution of benefits across generations (S. Wang & Lu, 2019). With additional challenges posed by financial market uncertainties and demographic shifts, innovative designs for pension plans and associated investment strategies are required to effectively manage risks and ensure intergenerational fairness.

Intergenerational risk sharing is a vital concept within pension economics. It pertains to the sharing of financial risks and resources among different age groups (Gordon & Roger, 1988). By distributing the uncertainties associated with pension funding - such as investment return volatility and unpredicted longevity - across multiple generations, the burden is not disproportionately carried by any single age cohort (Bégin, 2020). Longevity risk is one of the critical factors that need to be shared across generations. Essentially, longevity trends indicate that individuals born in later years tend to have a longer lifespan compared to those born earlier. Longevity risk refers to the uncertainty that disrupts people's predictions of these longevity trends. Over the past few decades, nations worldwide have been experiencing a demographic shift. This shift is marked by an increase in the average life expectancy at birth and a decrease in lifetime fertility rates (Hári et al., 2008; Pascariu et al., 2018). According to Gollier (2008), intergenerational risk sharing has the potential to smooth consumption patterns and hedge against unanticipated longevity risks, thereby advancing societal welfare and promoting equity. Furthermore, intergenerational risk sharing allows for the absorption of financial shocks over extended periods, encompassing the working and retirement phases of an individual's life. This temporal interaction is particularly crucial for pension schemes, which often span multiple decades and therefore involve more than one generation (Bégin, 2020).

To craft an efficient pension plan, it is crucial to understand two common types of pension schemes: Defined Benefit (DB) and Defined Contribution (DC). Guan and Liang (2014) stress the necessity of a balanced approach to intergenerational risk sharing within these plans. This balance aims to distribute risks and benefits fairly among different age cohorts, thus strengthening the sustainability of the pension system and averting excessive burden on any single generation. A deep understanding of the distinct features and risk allocation mechanisms of DB and DC plans is fundamental to this balancing act.

DB plans provide a specified pension payout at retirement, typically calculated based on the employee's salary history and length of service. These plans guarantee retirees a secure and predictable income stream, thereby mitigating the risk of income shortfall during retirement (Cocco & Lopes, 2011; Poterba et al., 2007). However, DB plans impose the bulk of the financial risk on the plan sponsor, usually the employer, who is obligated to ensure adequate funds to fulfill the promised benefits, irrespective of investment performance or longevity trends (Ali & Frank, 2019). Thus, DB plans result in a skewed intergenerational risk sharing, with the working generation shouldering the financial risk while the retiree generation benefits from the assured income.

On the other hand, DC plans dictate the contributions from the employer, employee, or both, without promising a specific retirement benefit. The retirement income under DC plans depends on the total value of contributions and the return on investments made with these funds. These plans offer flexibility and the potential for a higher retirement income if investments perform well (Poterba et al., 2007). Nonetheless, DC plans shift substantial investment and longevity risks onto individuals (Zelinsky, 2004). Thus, DC plans result in intergenerational risk sharing that burdens retirees with uncertainties associated with market fluctuations and the risk of outliving their pension savings.

Considering these observations, it becomes clear that both DB and DC pension schemes pose challenges concerning effective intergenerational risk sharing (Alonso-García et al., 2018). DB plans concentrate risk mainly on the working generation, while DC plans disproportionately burden retirees with risk. This unequal risk distribution highlights the shortcomings of these traditional pension models, necessitating a balanced and sustainable approach for intergenerational risk sharing in pension scheme design.

In response to these limitations of DB and DC plans, various countries globally have begun exploring Hybrid Pension Plans. These innovative schemes combine features of both DB and DC plans to achieve a balanced distribution of risks and benefits among stakeholders. By integrating risk-sharing mechanisms, Hybrid Pension Plans strive to uphold intergenerational fairness without unduly burdening any single generation. For instance, the Netherlands and Sweden have established hybrid systems that blend elements of DB and DC pensions. In the Netherlands, the

"Collective Defined Contribution" plan, which incorporates risk-sharing components, has gained considerable traction (Sorsa & van der Zwan, 2022). Similarly, Sweden's "Notional Defined Contribution" plan mimics a DC structure but integrates certain DB features, including sharing of longevity and investment risks among participants (Holzmann et al., 2019). In the United States, 'Cash Balance Plans' – a type of hybrid pension – promise a specified account balance at retirement, merging features of both traditional DB and DC plans (He et al., 2022). While these Hybrid Pension Plans vary in design, all share a fundamental commitment to balancing risks and benefits across generations, highlighting their potential to address challenges inherent in traditional pension schemes.

This chapter endeavors to investigate the potential of hybrid pension schemes, which aim to marry the best aspects of DB and DC plans. These hybrid schemes strive for a more balanced distribution of risks, facilitating intergenerational risk-sharing. Our research explores optimal investment strategies for collective hybrid pension plans using an Overlapping Generations (OLG) model – a tool introduced by Allais (1947) and Samuelson (1958) that encapsulates intergenerational risk-sharing mechanisms. Set within the Constant Relative Risk Aversion (CRRA) framework, our model simulates pension plan dynamics using the Monte Carlo method.

Our research also applies a Bayesian Optimization technique to pinpoint the optimal combination of parameters that maximize welfare while accounting for demographic, economic, and financial risks. By comparing the results of our model with traditional DB and DC plans, we hope to illuminate the potential of hybrid pension schemes to manage risks more effectively and promote intergenerational equity. This research is particularly pertinent for countries like China and Poland grappling with an aging population and a pressing need for financial market reforms. The theoretical framework established in this chapter provides the foundation for the empirical simulations detailed in Chapters 4 and 5. These chapters will apply and test the model within the specific demographic and economic contexts of China and Poland, demonstrating the practical application and potential benefits of hybrid pension schemes in addressing the challenges of aging populations and the need for pension system reform.

3.2. Literature Review

3.2.1. Intergenerational Risk Sharing

Intergenerational risk sharing represents a critical dimension in the design and assessment of financial markets and economic policies, addressing the inherent challenge of distributing risks and rewards not just within a generation but across multiple generations. The seminal work by Diamond (1977) laid foundational insights into the inefficiencies arising from the inability of current generations to engage in risk-sharing with unborn generations, highlighting the incomplete nature of competitive financial markets. This theme was further elaborated by Roger H. Gordon and Hal R. Varian in 1988, who explored the implications of these market inefficiencies for public finance and policy design, advocating for mechanisms that facilitate risk distribution across different age cohorts. The discussions by Ball and Mankiw (2001), and the influential contributions of Robert J. Shiller in 1999 and 2003, expanded the discourse by examining the role of financial instruments and market innovations in mitigating these inefficiencies. Shiller's work, in particular, emphasized the potential of financial innovation to bridge the gap in risk-sharing opportunities between generations.

3.2.2. Overlapping Generations Model

The Overlapping Generations (OLG) model, initially conceptualized by Maurice Allais and later formalized by Paul A. Samuelson in his 1958 paper "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money," stands as a cornerstone in economic theory, particularly in the analysis of pension systems, intergenerational transfers, and fiscal policy. Samuelson's work provided a foundation for understanding how economies operate over time, with successive generations of individuals interacting in markets for goods, services, and financial assets. This model has paved the way for a rich body of literature that explores the dynamics of economic behavior across different generations, highlighting the critical role of government in managing public pensions and ensuring intergenerational equity.

Following the foundational works, researchers have extensively applied the OLG model to study the intricacies of pension systems. In the mid-20th century, economists like Aaron (1966) and Diamond (1965) expanded upon Samuelson's model to include aspects of social security systems, demonstrating how these could be conceptualized as intergenerational transfers and examining their sustainability within growing economies. Diamond's inclusion of production into the OLG framework allowed for a more nuanced analysis of public pension schemes, sparking debates on the fiscal sustainability of such systems in the face of demographic shifts and economic changes.

In more recent years, the OLG model has been instrumental in exploring various economic phenomena, influencing the analysis of fiscal policy, human capital development, and monetary policy. For instance, Annabi et al. (2011) leverage the OLG model to assess the dynamic effects of government spending on post-secondary education in the context of Canada's aging population. They explore the repercussions of different financing strategies, including lump-sum taxes and income taxes, on human capital accumulation and economic welfare. Their findings underscore the critical role of funding mechanisms in shaping the long-term economic landscape, emphasizing the need for efficient educational spending to boost human capital and labor force growth.

Krueger et al. (2021) delve into the realm of optimal capital taxation within the OLG framework, addressing the complex interplay between precautionary savings due to uninsurable idiosyncratic labor income risk and the government's taxation policies. Their research contributes significantly to the discourse on tax policy design, balancing the need for savings against the backdrop of economic uncertainties.

Furthermore, Hu et al. (2023) examine conventional monetary policy through the lens of the OLG model, focusing on the asset-substitution channel between government debt and private assets. Their analysis reveals unique implications of monetary adjustments, challenging the conventional wisdom derived from New Keynesian models and advocating for a nuanced understanding of monetary policy's real and nominal effects.

The OLG model's application to pension systems reveals insightful perspectives on demographic changes, system reforms, and their socio-economic implications. Initially, Aglietta et al. (2007) delve into the critical need for reform within Pay-As-You-Go (PAYG) systems, addressing the challenges posed by an aging population and increased life expectancy. Utilizing the INGENUE world model, their analysis simulates the macroeconomic effects of pension reforms in Europe, evaluating scenarios like Constant Contribution Rate (CCR), Postponing Retirement Age (PRA), and Constant Gross Replacement (CGR). Their findings underscore the significant impact of pension reforms on savings, investment, and global economic dynamics, emphasizing the importance of considering financial globalization and demographic changes in ensuring sustainable and equitable pension systems. Building on this foundational analysis, Cipriani (2014) further emphasizes the urgency of pension system reform, spotlighting the sustainability threats due to demographic shifts. This sequence of discussions presents a comprehensive overview of the OLG model's utility in addressing the multifaceted challenges pension systems face, from demographic pressures to the broader economic implications of reform strategies.

Building upon these demographic insights, Cui et al. (2011) and Tyrowicz et al. (2018) provide a nuanced examination of pension system reforms and designs. Cui et al. (2011) focus on the benefits of intergenerational risk sharing (IRS) within funded pension systems, proposing a framework for optimizing pension design parameters to enhance welfare through balanced risk and return. This approach is complemented by Tyrowicz et al. (2018), who explore the socioeconomic impacts of transitioning to defined contribution schemes, particularly in terms of inequality. They suggest that while such shifts may introduce greater consumption inequality, the strategic use of minimum pension benefits could mitigate these adverse effects.

Wang (2021) extends the analysis to notional defined contribution (NDC) pension schemes, examining their potential to positively affect unemployment and fertility rates. By highlighting how improved NDC return rates could incentivize fertility and support pension system sustainability, Wang connects structural pension reforms to broader economic and demographic trends. This comprehensive discussion, anchored in the OLG model, illustrates the model's critical role in guiding sustainable and equitable pension reform efforts, aligning structural adjustments with demographic realities and socioeconomic objectives.

3.3. Model Formulation

The model we employ to explore optimal investment strategies for a collective hybrid pension plan is an Overlapping Generations (OLG) model. This model captures the life cycle of individuals who live, work, and retire, thus creating a continually evolving population structure. This model, first proposed by Allais (1947) and Samuelson (1958), provides a powerful tool for analyzing intergenerational equity and risk-sharing.

Consider a pension plan with a working population and a retired population. Each individual starts working at age E and retires at age R. The pension plan has a constant contribution rate p and a constant benefit rate b. The asset returns follow a stochastic process. We use mortality rates to calculate survival probabilities for each individual. The goal of the pension plan simulation is to find the optimal combination of the contribution rate p, risk-allocation parameters α and β, and risky asset investment proportion w that maximize the aggregate utility of the consumptions over the entire simulation period:

$$U = \max_{\{p,\alpha,\beta,\omega\}} E_0 \left[\int_0^T e^{-\delta t} \frac{c_t^{1-\gamma}}{1-\gamma} dt \right] \tag{3.1}$$

The model includes several parameters that capture the key features of the pension system and the economic environment. These parameters are presented in Table 1.

Table 3.1. List of Notation

Notation	Description
E	Age of entry
R	Age of retirement
M	Maximum age
G	Number of overlapping generations, $G = M - E$
W	Number of working generations
P	Number of retired generations
q_x	Mortality rate at age x
p_t	Contribution rate at time t
b_t	Benefit rate
p	Target contribution rate
b	Target benefit rate

α	Absorb speed of imbalance of fund of contribution rate
β	Absorb speed of imbalance of fund of replacement rate
A_t	Fund asset at time t
L_t	Fund liability at time t
i_t	Fund investment return at time t
r	Risk-free rate
μ_t	Risky asset return at time t
w	Weight investment in risky asset
γ	risk aversion parameter

Source: Own Presentation

3.3.1. Demographic

The demographic structure of the model population is constructed in Chapter 2. In our model, the entry age E and maximum age M are determined by drawing from literature on OLG models and pension scheme design (see Bégin, 2020; Bucciol et al., 2017; D. H. J. Chen et al., 2016; Cui et al., 2011). Specifically, individuals are set to enter the model at age E and can live up to a maximum age of M.

The demographic model under consideration assumes a fixed maximum lifespan M and the entry age E. For each individual, their lifespan is considered to be a random variable, T, taking values between the entry age and the maximum lifespan. In the context of our model, time t represents the number of years since an individual entered the model (i.e. since they turned age E). The model takes into account survival rates and mortality rates to characterize the demographic structure. For an individual aged x, the future lifetime is denoted as T_x, and it ranges from 0 to $M - x$. The survival function, $S_x(t)$, represents the probability that the future lifetime T_x exceeds t, given that the individual is currently alive at age x :

$$S_x(t) = P[T_x > t] = 1 - F_x(t) = P[\, T > x + t \mid T > x \,] = \frac{S(x+t)}{S(x)} \tag{3.2}$$

where $F_x(t)$ is the cumulative distribution function of the future lifetime T_x for an individual currently aged x. This function gives the probability that the future lifetime T_x is less than or equal to t :

$$F_x(t) = P[T_x \leq t] = P[\, T \leq x + t \mid T > x \,] = \frac{S(x) - S(x+t)}{S(x)} \tag{3.3}$$

Here, $S(x)$ represents the survival probability at age x - the probability that an individual's lifetime T exceeds x, and $S(x+t)$ represents the survival probability at age $x+t$.

The probability of death within a specific period can be expressed as follows:

$${}_t q_x = F_x(t) = \int_0^t f_x(s)ds \tag{3.4}$$

This equation represents the probability of death at time t for an individual aged x, which is equal to the integral of the density function of the future lifetime T_x over the interval from 0 to t.

Inspired by the work of Heer and Maußner (2009), we construct the population model as follows. A period, t, corresponds to one year. At each period t, a new generation of workers is born, where newborns have a real-life age of E, denoted as $s = 1$. All generations retire at the end of age $s = T^W = \mathrm{R} - \mathrm{E}$ and live up to a maximum age of $s = T^W = \mathrm{M} - \mathrm{E}$. The number of periods during retirement is equal to $T^P = G - T^W = \mathrm{M} - \mathrm{R}$.

Let $N_t(s)$ denote the number of people of age s at time t, and N_t the total population at time t. In period t, the newborn cohort grows at a rate n_t. All individuals of age s survive until age $s+1$ with a probability q_t^s, where $S_t^0 = 1$ and $S_t^G = 0$. For the steady state, we assume the population growth rate is constant, i.e., $n_t = n$. The growth of the population is then defined by:

$$N_t(1) = (1 + n_t)N_{t-1}(1) \tag{3.5}$$

and

$$N_t(s) = S_t^{s-1} N_{t-1}(s-1) \tag{3.6}$$

We define the measure of the s-year old in period t, μ_t^s, as:

$$\mu_t^s \equiv \frac{N_t(s)}{N_t} \tag{3.7}$$

By definition, the sum of all measures μ^s is equal to one:

$$\sum_{s=1}^{G} \mu_t^s = 1 \tag{3.8}$$

Then

$$\mu^{s+1} = \frac{S^s}{1+n}\mu^s \tag{3.9}$$

3.3.2. Financial Market

Followed by Cui et al. (2011), the financial market consists of a risk-free asset and a risky asset. We assume that the non-stochastic interest rate is represented by r. The stock prices follow a geometric Brownian motion characterized by a constant drift μ, σ is is the volatility of stock returns. The investment portfolio comprises a combination of risk-free assets and stocks, where the portfolio weight assigned to stocks is denoted by ω. The dynamic of a portfolio is described:

$$\frac{dW_t}{W_t} = [r + \omega(\mu - r)]dt + \omega_t \sigma dZ_t \tag{3.9}$$

To exclude arbitrage opportunities, we assume that $\mu > r$. We operate under the assumption that the pension plan is restricted from borrowing funds and faces short-sale constraints, which necessitates that the value of ω is bounded within the range $0 \leq \omega \leq 1$.

3.3.3. Pension Plan Dynamic

Liability

In the scheme we are considering, liabilities bear a resemblance to those in DB plans. This implies that variations in asset returns can induce a mismatch risk between the assets and liabilities of the fund. Consequently, we commence our analysis with a traditional DB scheme, setting a target liability denoted as L, a target contribution rate represented as p, and a target benefit labeled as b.

Particularly in an average salary DB scheme of the traditional type, the retirement benefit corresponds to a fixed proportion of the earnings from labor obtained during the period of employment. Supposing that the active member receives a (real) labor income that is normalized to 1 during this period, the target benefit b could be construed both as a fraction and as a concrete monetary value. These benefits are primarily financed through contributions and returns from investments. In general terms, it can be inferred that as b ascends, p should follow a similar upward trajectory.

$$\mathbb{E}^{\mathbb{Q}}\left[\sum_{t=0}^{R-E-1} e^{-rt}p_t\right] = \mathbb{E}^{\mathbb{Q}}\left[\sum_{t=R-E}^{M-E-1} e^{-rt}b_t\right] \tag{3.10}$$

Assuming further that the predetermined benefit and contribution levels are set such that the expected value under risk-neutral probability measure $\mathbb{Q}$ of the benefit level b_t and contribution level p_t at any given time t is equivalent to their target levels b and p, respectively:

$$\mathbb{E}^{\mathbb{Q}}[b_t] = b \text{ and } \mathbb{E}^{\mathbb{Q}}[p_t] = p \tag{3.11}$$

This gives us an equation similar to the one derived by Cui et al. (2011), which states that the present value of contributions equals the present value of benefits:

$$\sum_{t=0}^{R-E} e^{-rt}p = \sum_{t=R-E}^{M-E} e^{-rt}b \tag{3.12}$$

Then, we build the liability for each age cohort, l_x which is calculated as the discrepancy between the present value of guaranteed benefits and the present value of the forthcoming risk-free contributions.

$$l_x = \begin{cases} \int_R^M e^{-r(t-x)} b dt - \int_x^R e^{-r(t-x)} p dt, & \text{for } x < R \\ \int_x^M e^{-r(t-x)} b dt, & \text{for } R < x < M \end{cases} \tag{3.13}$$

Where $represents$ the specific age cohort, for cohorts below retirement age, the liability is the difference between the present value of guaranteed benefits and future risk-free contributions. For those beyond the retirement age, the liability equals the present value of guaranteed benefits.

When incorporating the survival model, the calculation of liability takes into account the probability of a member's survival at each age. This effectively measures benefits and contributions based on the actual population distribution for each generation. The simplified expression of aggregated liability is

$$L = \frac{b}{r^2}(e^{-rT} - e^{-rR} + rT - rR) \sum_{i=1}^{W} \mu_i^s + \frac{p}{r^2}(1 - e^{-rR} - rR) \sum_{i=W}^{G} \mu_i^s \tag{3.14}$$

Asset

Let A_t denote the time- t value of the pension fund's assets, commencing with an initial asset value defined as $A_0 = L$. Here, ω signifies the proportion of assets invested in riskier ventures. This portfolio weight, ω, is constant over time and applies uniformly to every cohort within the fund. Accordingly, the dynamics of the fund's assets are determined by these parameters:

$$dA_t = \left[A_t\big(r + \omega(\mu - r)\big) + Wp_t - Pb_t\right]dt + \omega\sigma A_t dZ_t \tag{3.15}$$

This equation captures the interplay between market returns on the portfolio, the incoming cash flows from the contributions, and outgoing cashflows in the form of benefits.

The surplus S, which represents the difference between the updated assets and the liability, is then computed as $S = A - L$ Based on the current surplus status, the contribution rate and the benefit rate for the subsequent year are updated. Specifically, in the event of a positive surplus, the contribution rate, pt, decreases, and the benefit rate, bt, increases. Conversely, in the event of a negative surplus, the contribution rate increases, and the benefit rate decreases. These adjustments are captured by:

$$\begin{aligned} pt &= p - \alpha * \frac{S}{(W * \sum_{i=1}^{W} \mu_i^s)} \\ bt &= b + \beta * \frac{S}{\left((M - R) * \sum_{i=W}^{G} \mu_i^s\right)} \end{aligned} \tag{3.16}$$

The parameters α and β determine the sensitivity of these adjustments to the surplus. When the sum of $a + b$ is near the risk-free rate r, each generation within a given period solely accrues 'interest' on their remaining funds, while the principal is carried forward indefinitely. To ensure temporal stability (that is, to avoid unbounded growth in surplus values), it is necessary to impose the condition $\alpha + \beta > r$.

Table 3.2. Summarize Table of Pension Schemes Type

Defined Benefit Scheme (DB)	$\alpha > 0$	$\beta = 0$
Defined Contribution Scheme (DC)	$\alpha = 0$	$\beta > 0$
Hybrid Scheme	$\alpha > 0$	$\beta > 0$

Source: Cui et al. (2011)

3.3.4. Risk Preference

The decision-making in pension plan design often involves trade-offs between risk and return, which requires an understanding of the risk preferences of the stakeholders. A common model used to represent risk preferences is the Constant Relative Risk Aversion (CRRA) utility function, which reflects a consistent level of risk aversion regardless of the level of wealth or income.

The CRRA utility function is defined as:

$$U(c) = \frac{c^{(1-\gamma)}}{1 - \gamma} \tag{3.16}$$

We assume that G cohorts are homogeneous in nature, meaning they have equivalent population sizes and identical preferences. Each individual within these cohorts employs a CRRA utility function.

We assume that prior to retirement, individuals set aside a portion of their income, denoted as p_t, to contribute towards their pension plan. Consequently, the disposable income available for their consumption reduces to the remainder. Post-retirement, individuals' consumption patterns are dictated exclusively by the pension benefits they receive, denoted as b_t. As there are no further contributions made to the pension plan during this phase, the entirety of the pension benefits is allocated for their consumption. Therefore, the consumption pattern is described:

$$\begin{aligned} c_t &= 1 - p_t, t < R \\ c_t &= b_t, R \leqslant t < M \end{aligned} \tag{3.17}$$

3.3.5. Welfare Evaluation

Drawing inspiration from the work of Cui et al. (2011), a methodology also adopted by subsequent researchers such as Chen et al. (2016) and Bégin (2020), we utilize the measure of certainty equivalent consumption (CEC) to evaluate the performance of collective pension schemes. The CEC offers a direct approach to quantifying welfare, which can be computed based on the subsequent equation:

$$U = \int_0^T e^{-\delta t} \frac{(CEC)^{1-\gamma}}{1-\gamma} dt \tag{3.18}$$

3.3.6. Simulation Process

The Pension Policy Optimization problem aims to maximize the lifetime utility of newly entered participants in the pension plan. This objective is represented by the following utility function:

$$U = \max_{\{p,\alpha,\beta,\omega\}} E_0 \left[\int_0^T e^{-\delta t} \frac{c_t^{1-\gamma}}{1-\gamma} dt \right] \tag{3.19}$$

We implement Monte Carlo simulations and Bayesian Optimization to address the collective schemes. Initially, we define a parameter space for our decision variables across four dimensions. Using Bayesian Optimization, we explore this parameter space more efficiently than with a traditional grid search. This method evaluates the scheme's performance across various parameter combinations by learning from previous simulations. After an initial round of optimization, the parameter space is refined around the best-found parameters to fine-tune the optimization process. For each scenario, we simulate the asset trajectory over time, leading to a specific funding surplus or deficit. Based on these results, we dynamically adjust the contributions and benefits, denoted as p_t and b_t, in accordance with the risk-sharing rules defined in the equations.

Subsequently, we evaluate the welfare function's outcome for the incoming cohort. Through Bayesian Optimization, we efficiently locate the global maximum of the welfare function and identify the corresponding optimal parameter values $\{p, \alpha, \beta, \omega\}$. This approach enhances our ability to fine-tune the decision parameters to achieve our objective function's goals, leveraging the predictive power of Bayesian Optimization to navigate the parameter space towards optimal solutions. We implement the simulation process using Python, and partial codes are provided in Appendixes.

3.4. Conclusion

In conclusion, Chapter 3 delves into the potential of hybrid pension schemes to harmoniously combine the strengths of Defined Benefit (DB) and Defined Contribution (DC) plans. This analysis is driven by the pressing need to address increasing life expectancies and the aging global population, which place considerable stress on traditional pension systems. By integrating features from both DB and DC models, hybrid pension plans aim to distribute risks and benefits more equitably across generations, potentially enhancing the sustainability of pension systems against the backdrop of demographic and economic uncertainties.

Our exploration employs an Overlapping Generations (OLG) model, enhanced with a Constant Relative Risk Aversion (CRRA) framework, to simulate and analyze the dynamics of these hybrid plans. The Monte Carlo method, supplemented by Bayesian Optimization, enables us to identify optimal investment strategies that balance the trade-off between risk and reward, thus maximizing welfare while considering demographic, economic, and financial risks. The insights garnered from this analytical approach will lay a solid foundation for empirical tests in Chapters 4 and 5, where the model's applicability will be examined within the specific socio-economic environments of China and Poland.

Ultimately, this chapter aims to illuminate how hybrid pension schemes can provide a viable solution to the challenges posed by modern demographic trends and financial market volatility. By offering a more balanced approach to risk sharing and intergenerational equity, hybrid plans may hold the key to enhancing the financial security and well-being of both current and future generations in various global contexts.

Chapter 4. Optimal Investment Strategies-China Simulation

4.1. Introduction

Chapter 4 embodies a critical juncture in our investigation into pension systems, spotlighting the specific challenges and opportunities presented by China's demographic and regulatory environment. This segment of the dissertation transitions from theoretical abstraction to empirical application, underscoring the practicality of the hybrid pension model within the complex framework of China's evolving population dynamics and pension policies. This chapter is not merely theoretical; it is a pivotal endeavor aimed at narrowing the divide between conceptual models and their practical utility, illuminating the tangible implications of pension system reforms tailored to China's unique demographic trends and policy landscape.

The chapter initiates with a detailed exposition of the simulation's methodological framework, which includes recalibrating demographic trends informed by the Lee-Carter method projections elaborated in Chapter 2. This process is critical for ensuring the simulation's relevance to China's current and foreseeable demographic shifts. Furthermore, the analysis meticulously incorporates an evaluation of both existing and prospective regulatory modifications affecting China's pension policy framework. The objective of this simulation is to furnish a comprehensive analysis that elucidates the interplay between demographic changes and regulatory frameworks and their collective impact on devising and refining investment strategies under the aegis of collective hybrid pension plans. The ultimate goal is to equip stakeholders with solid, evidence-based recommendations for developing effective pension schemes. We assess pension performance using Certainty Equivalent Consumption (CEC), a measure that quantifies the consistent consumption level providing the same utility as the expected variable consumption from the pension plan. This method directly evaluates how well pension schemes support retirees' quality of life.

An integral component of this chapter is the rigorous examination of our model's robustness, achieved through a series of simulations tailored to the Chinese context. This robustness testing is pivotal, as it not only validates the model's resilience and adaptability to China's specific pension system challenges but also ensures the reliability of the proposed investment strategies in the face of demographic and policy variability. By subjecting our model to this rigorous evaluation, we aim to underscore its utility and reliability, reinforcing the model's capacity to inform pension policy development and strategic investment planning in an ever-changing socio-economic landscape.

In essence, Chapter 4 serves as a bridge connecting theoretical insights to practical applications within the realm of pension systems, particularly within the context of China. By leveraging detailed simulations and robustness tests, this chapter contributes significantly to the discourse on pension sustainability, offering a nuanced understanding of how demographic trends and regulatory policies shape the effectiveness and viability of pension investment strategies. Through this analytical journey, we aim to provide policymakers, practitioners, and scholars with a robust framework for navigating the complexities of pension system reform and optimization, ensuring the long-term sustainability and efficacy of pension schemes in accommodating the needs of diverse and aging populations.

4.2. Assumption

Age of Entry (E)

According to the Ministry of Human Resources and Social Security of the People's Republic of China (2018), the minimum working age is established at 16 years. Furthermore, documentation from Gov.cn (2014) specifies that the entry age for the Urban and Rural Residents Pension Scheme (URRPS) is also 16 years old. Consequently, for the purposes of this analysis, the age of entry into the pension scheme is assumed to be 16 years.

Age of retirement (R)

Gov.cn (2014) indicates that the retirement age for participants in the Urban and Rural Residents Pension Scheme (URRPS) is standardized at 60 years for both females and males. Thus, in the context of this research, the age of retirement is presumed to be 60 years.

Weight investment in risky asset (w)

According to guidelines published by Gov.cn (2015), the allocation towards risky assets such as equities, equity funds, hybrid funds, and equity-based pension products is capped at 30% of the total net asset value of the pension fund. Therefore, for this study, the weight of investment in risky assets (ω) is constrained within the range of 0 to 0.3 ($0 \leq \omega \leq 0.3$).

Maximum Age (M)

Building on the analysis presented in Chapter 2, this study incorporates mortality data sourced from the China Population and Employment Statistics Yearbook, as published by the National Bureau of Statistics (2023). The data encompasses age groups from 0 to 89, with a categorization for ages 90 and above. Consequently, for the purposes of this simulation, we have established the maximum age parameter at 90.

Employer Contribution

In accordance with the guidelines set forth by the Ministry of Human Resources and Social Security of the People’s Republic of China (2012), the basic pension insurance system in China operates on a contributory basis. As mandated by the state, employer contributions to the basic pension insurance should not exceed 20% of the total payroll. Typically, the contribution rate for employers is fixed at 20% (Liu & Sun, 2016). Based on this regulatory framework, our analysis will proceed under the assumption that the employer contribution rate to employees' wages is consistently 20%.

4.3. Numerical Results

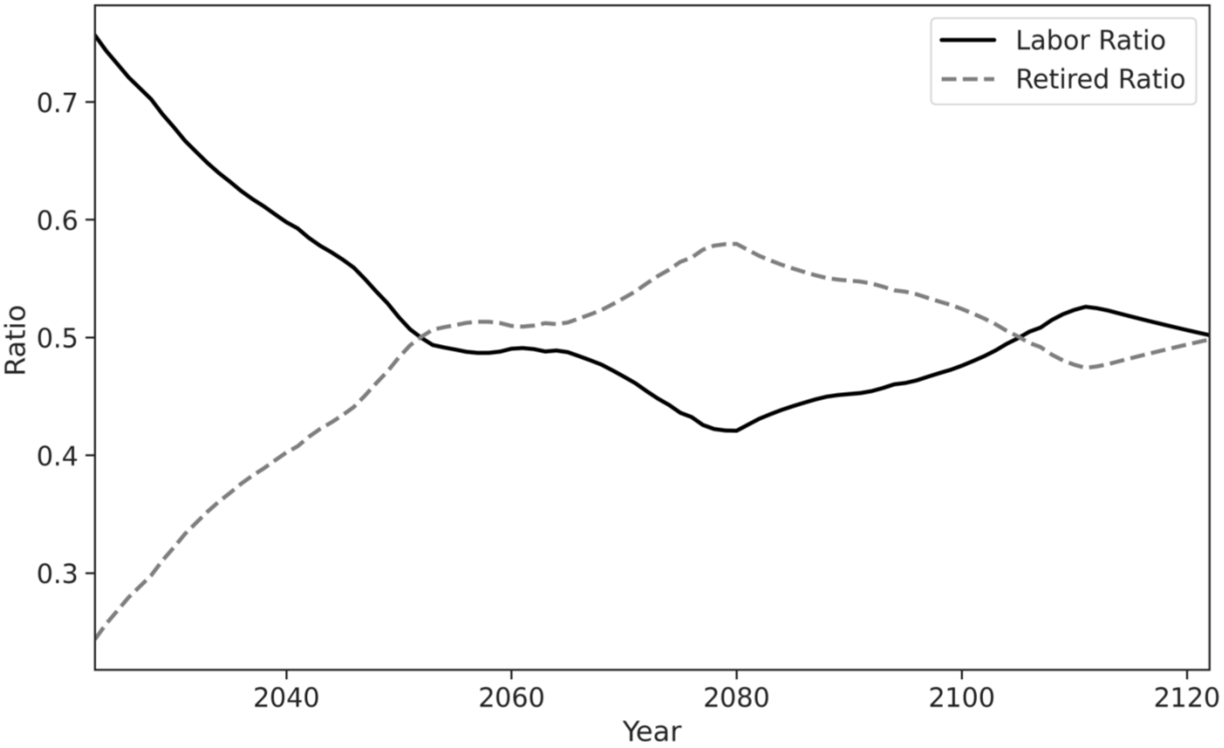
Grounded in previous academic literature, we have made specific selections for the default values of risk-free and risky assets. A real risk-free interest rate r of 0.02 is proposed. We project the risky asset to yield an average return μ of 0.06, implying an equity risk premium of 4%. This premium concurs with the estimation of the long-standing equity premium by Fama and French, (2002). Followed by Cui et al. (2011), the volatility δ is assumed to be 15%. And the risk aversion coefficient γ of 5. All schemes are initially fully funded (i.e. $A_0 = L$).

In this chapter, we employ the two-period Overlapping Generations (OLG) model to assess the economic impacts of different birth rates on the labor market, using three scenarios: low (6 per 1,000), moderate (10 per 1,000), and high (14 per 1,000). These scenarios are based on historical birth rates from China and Poland between 1997 and 2022, which fluctuated between 6.0 and 17.0 births per 1,000 people, as outlined in Chapter 2. By segmenting the human lifecycle into two principal stages—'young', spanning ages 16 to 60, where individuals contribute to the workforce, and 'old', covering ages 61 to 90, signifying retirement—the model facilitates a precise division between the economically active and inactive population segments. This bifurcation is instrumental in evaluating the repercussions of birth rate fluctuations on the labor market. Consequently, we introduce the concept of the labor ratio, defined as the share of the working-age populace (16 to 60) relative to the aggregate population under consideration (16 to 90). In parallel, the retired ratio is determined as the share of individuals aged 61 to 90 within the same demographic scope. By applying these ratios, we examine how birth rate variations affect labor market dynamics in China, with Poland's analysis to follow in the next chapter. This consistent application of birth rate variants ensures a coherent approach across both case studies.

4.3.1. Low Birth Rate

Figure 4.1 displays the projected labor and retired ratios over a century, set against the backdrop of a low birth rate. The labor ratio, depicted by the solid line, begins above 0.7, signifying a larger working-age population in comparison to the retired segment. This ratio experiences a consistent decline, falling below 0.5 around the year 2090. There is a modest increase in the labor ratio following this dip, which peaks around 2110 before it descends once more, finishing slightly below 0.6 as we approach 2120. In contrast, the retired ratio, shown with a dashed line, trends upwards, inversely correlating with the labor ratio. Commencing at a level significantly lower than the labor ratio, it rises and surpasses it near 2090, reflecting an expanding retiree cohort relative to those of working age. The peak of the retired ratio occurs shortly before 2080, after which it starts to decrease. These projections portend a shifting demographic structure, substantially affecting the labor market and pension system sustainability. The decade straddling 2080 to 2090 is particularly pivotal, marking a demographic shift where retirees are anticipated to outnumber the active workforce, posing challenges to the maintenance of retirement benefits and economic support structures.

Figure 4.1. China-Projections of the Labor Ratio and Retired Ratio over the Next 100 Years under Conditions of Low Birth Rate



Data Source: Own Presentation

Table 4.1 presents the optimized parameters for DC, DB, and Hybrid pension schemes under conditions characterized by a low birth rate. The DC scheme specifies a target contribution rate (p) of 0.2173, a high adjustment rate for the replacement rate ($\beta = 0.7280$), a conservative approach towards investment in risky assets ($\omega = 0.0341$) and sets a target benefit rate (b) at 1.3049. This configuration results in a CEC of 0.7000. The DB scheme, with a notably lower target contribution rate of 0.0128 and an adjustment rate for contribution rate imbalances (α = 0.0520), allocates a more significant portion to risky assets ($\omega = 0.2200$) and aims for a target benefit rate of 0.6655, leading to a CEC of 0.6938. Meanwhile, the Hybrid model, incorporating elements from both DC and DB schemes, sets its target contribution rate at 0.2496, with adjustment rates for fund imbalances ($\alpha = 0.0146, \beta = 0.7929$), an intermediate level of risky asset investment ($\omega = 0.0570$), and a target benefit rate of 1.4060, achieving the highest CEC of 0.7126. This indicates that, under low birth rate scenarios, the Hybrid scheme potentially offers a slightly superior welfare outcome compared to the standalone DC and DB models, as evidenced by its CEC value.

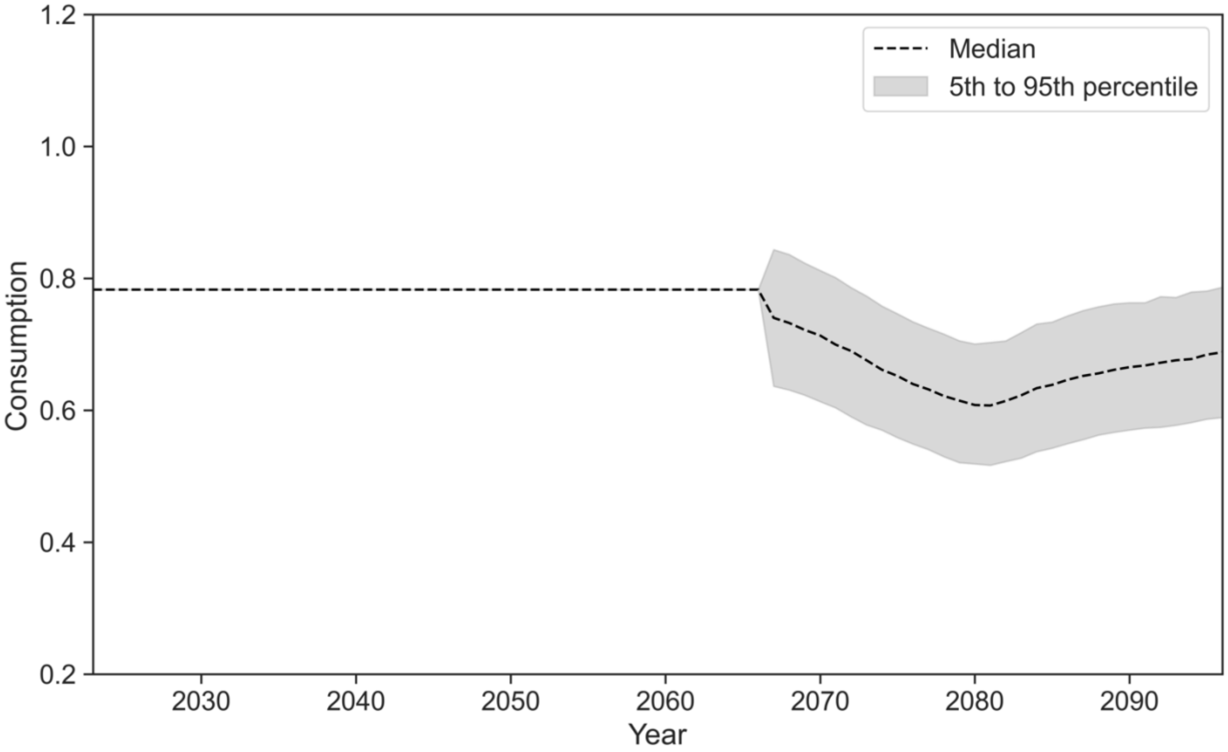
Table 4.1. China - Optimal Pension Schemes under Conditions of Low Birth Rate

Pension Scheme	DC	DB	Hybrid
p	0.2173	0.0128	0.2496
α	-	0.0520	0.0146
β	0.7280	-	0.7929
ω	0.0341	0.2200	0.0570
b	1.3049	0.6655	1.4060
CEC	0.7000	0.6938	0.7126

Data Source: Own Presentation

The study utilizes 5,000 Monte Carlo simulations to evaluate the consumption patterns of individuals who entered the workforce in 2022, tracking these patterns over a period of 74 years (from age 16 to 90). This evaluation focuses on identifying optimal pension strategies within the context of China’s low birth rate scenario. Figure 4.2 presents a detailed consumption profile for individuals within the optimal DC Scheme in China under a scenario of low birth rate, spanning from 2023 to 2096. Initially, the profile maintains a remarkable uniformity, with the 5%, 50% (median), and 95% quantiles all fixed at a median consumption level of 0.7827. However, a significant shift is observed as individuals transition into retirement, starting in 2067. At this juncture, median consumption begins to decline, signaling a departure from the pre-retirement stability. This decline is more pronounced by 2080, where median consumption drops to around 0.60, reaching its lowest point, and indicates the impact of demographic shifts, particularly the low birth rate, on the pension scheme's ability to sustain post-retirement consumption levels.

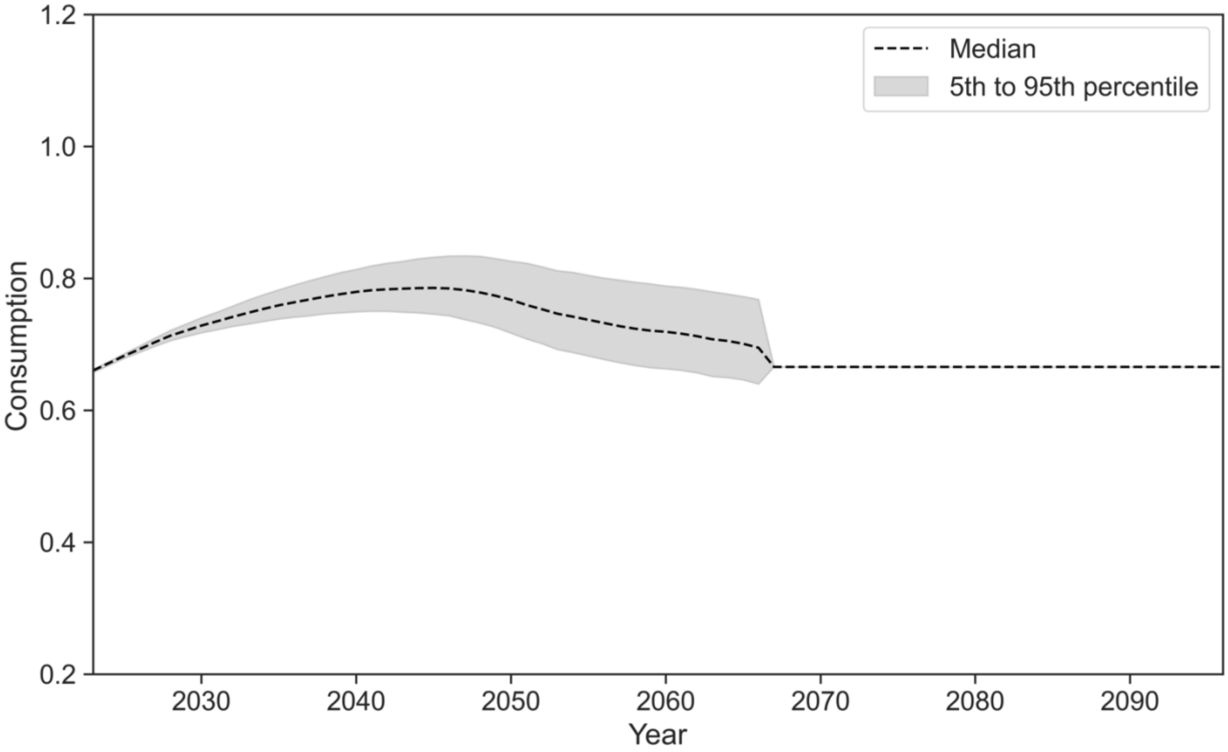
Figure 4.2. China - Consumption Profile within Optimal DC Scheme under Low Birth Rate Scenario



Data Source: Own Presentation

Figure 4.3 delves into the consumption patterns under the optimal DB scheme, providing a nuanced understanding of how median consumption evolves from the start of employment to retirement under China's low birth rate scenario. Initially, an upward trend in median consumption is observed, culminating in a peak around 2050 at approximately 0.76. Following this peak, there's a noticeable decline, with median consumption stabilizing at an average of about 0.65, which is maintained until the retirement age. Post-retirement consumption levels out at approximately 0.65, indicates the DB scheme's capacity to provide consistent benefits to its retirees, showcasing its inherent stability and predictability.

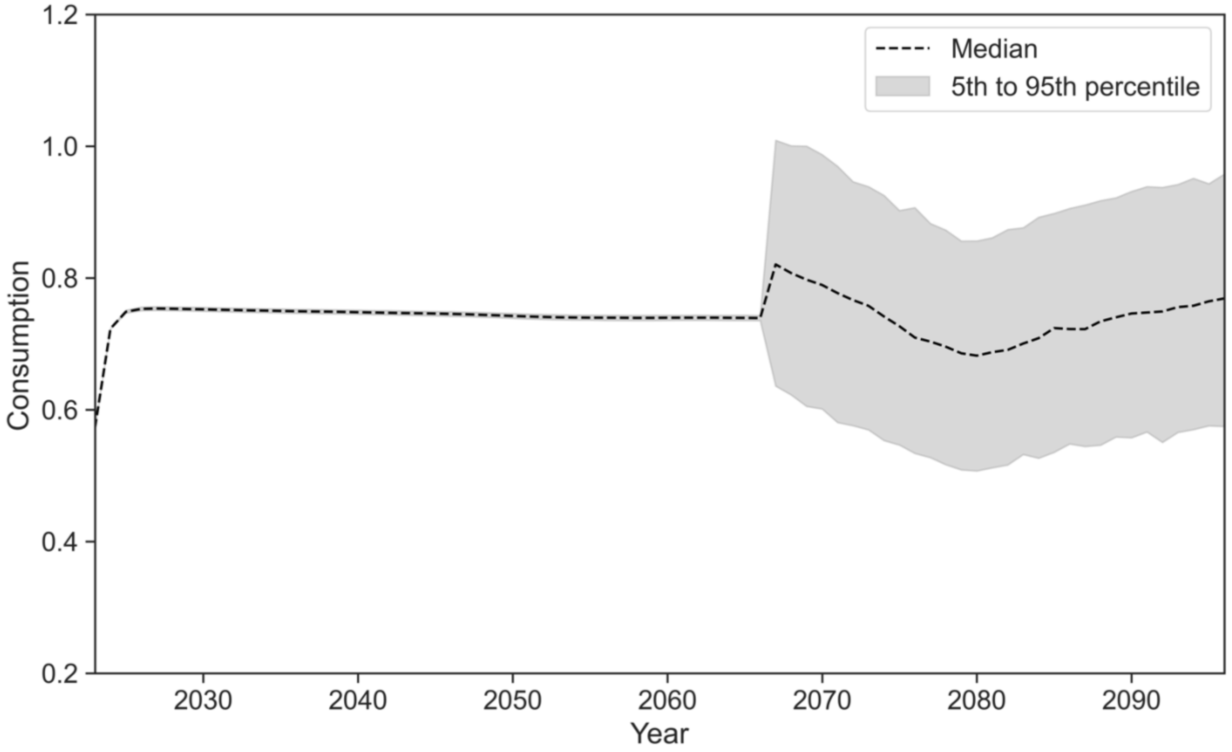
Figure 4.3. China - Consumption Profile within Optimal DB Scheme under Low Birth Rate Scenario



Data Source: Own Presentation

Figure 4.4 sheds light on the consumption patterns within the optimal Hybrid pension scheme, underlining its unique positioning between the DC and DB schemes in the context of China’s low birth rate scenario. Similar to the DC scheme, the Hybrid scheme demonstrates a period of stability in median consumption prior to retirement, marked by a relatively narrow quantile spread. This characteristic points to the scheme's effective management of fund imbalances through low adjustment rates, which contributes to a consistent standard of living for participants during their working years. In the pre-retirement phase, the Hybrid scheme maintains a median consumption level of about 0.78, akin to the DC scheme, but it distinguishes itself with a slight increase to just over 0.84 at the commencement of retirement. This uptick reflects the scheme's design to balance growth and security, leveraging the strengths of both DC and DB models to provide a smooth transition into retirement. As individuals progress through their retirement years, median consumption gently decreases to approximately 0.75, illustrating a managed decline that ensures a sustainable quality of life. The Hybrid scheme's performance underlines its effectiveness in delivering a balanced and pragmatic approach to pension planning under the low birth rate scenario. By combining elements of both DC and DB schemes, it achieves a commendable balance between providing stability and accommodating growth, thus minimizing the risk of abrupt consumption declines post-retirement.

Figure 4.4. China - Consumption Profile within Optimal Hybrid Scheme under Low Birth Rate Scenario

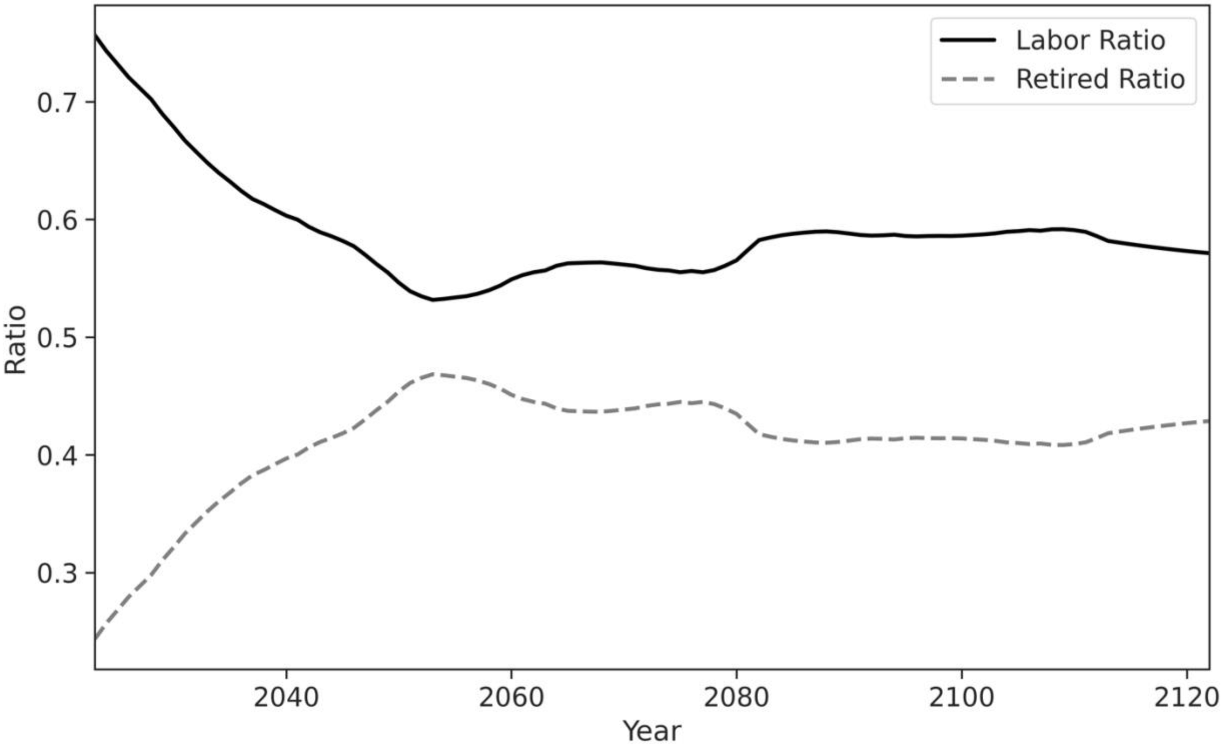


Data Source: Own Presentation

4.3.2. Moderate Birth Rate

Figure 4.5 presents an insightful perspective on China's demographic trends under conditions of a moderate birth rate over the next century. Contrasting with the low birth rate scenario, the moderate birth rate environment showcases a consistently higher labor ratio compared to the retired ratio. This dynamic serves to somewhat alleviate the pressures on the pension scheme by ensuring a relatively larger working-age population supports the retired cohort. The labor ratio exhibits a declining trend from 2022, reaching its lowest point around 2055, after which it stabilizes at approximately 0.68. Conversely, the retired ratio mirrors an inverse trend, highlighting the growing proportion of retirees over time.

Figure 4.5. China - Projections of the Labor Ratio and Retired Ratio over the Next 100 Years under Conditions of Moderate Birth Rate



Data Source: Own Presentation

Table 4.2 delineates the optimal parameters for DC, DB, and Hybrid pension schemes under the scenario of a moderate birth rate in China. This table presents a comprehensive overview of each scheme's target contribution rate (p), adjustment rates for contribution rate imbalances (α) and replacement rate imbalances (β), the proportion of investments in risky assets (ω), target benefit rate (b), and the resulting Certainty Equivalent Consumption (CEC). In the DC scheme, a target contribution rate of 0.3046, coupled with a moderate investment in risky assets ($\omega = 0.0818$) and an adjustment for replacement rate imbalances ($\beta = 0.0756$), culminates in a CEC of 0.8043. This indicates a solid welfare outcome for participants under the DC scheme. For the DB scheme, the notably low target contribution rate of 0.01891 and a higher allocation to risky assets ($\omega = 0.1273$), alongside an adjustment rate for contribution rate imbalances ($\alpha = 0.02192$), achieve a CEC of 0.7357. This reflects a more conservative welfare outcome compared to the other schemes. The Hybrid model, with a balanced approach incorporating a target contribution rate of 0.1575, adjustments for both types of fund imbalances ($\alpha = 0.0268, \beta = 0.3418$), and a similar level of risky asset investment ($\omega = 0.0830$) as the DC scheme, achieves the highest CEC of 0.8217. This suggests that the Hybrid scheme provides the best welfare outcomes among the three, highlighting its efficacy in blending the strengths of both DC and DB schemes to navigate the moderate birth rate scenario effectively.

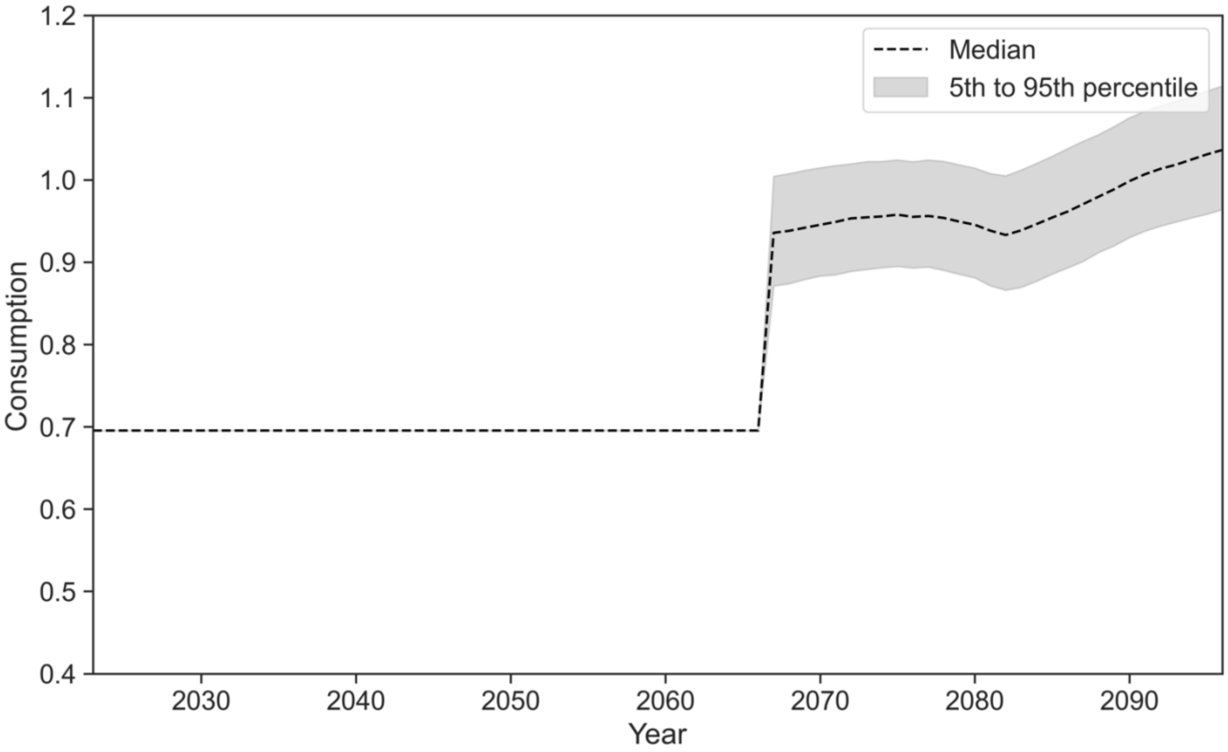
Table 4.2. China - Optimal Pension Schemes under Conditions of Moderate Birth Rate

Pension Scheme	DC	DB	Hybrid
p	0.3046	0.01891	0.1575
α	-	0.02192	0.0268
β	0.0756	-	0.3418
ω	0.0818	0.1273	0.0830
b	1.5777	0.6845	1.1178
CEC	0.8043	0.7357	0.8217

Data Source: Own Presentation

The consumption profile within the DC scheme, as captured in Figure 4.6, offers a distinct perspective on how individuals under a moderate birth rate scenario. Prior to retirement, the scheme provides a consistent median consumption level of approximately 0.7, demonstrating stability during the working years. This stability is a key feature of the DC scheme, reflecting the accumulation phase where individuals contribute to their pension while managing their consumption levels closely.As individuals transition into retirement, there is a notable shift in the consumption pattern. Median consumption begins to rise significantly, marking an improvement in living standards attributed to the commencement of pension benefit payouts. The increase continues progressively, with median consumption reaching around 0.95 shortly after retirement. This upward trend extends further until it peaks at a median consumption level of 1.05 by the maximum age considered in the study. Such an increase post-retirement is indicative of the DC scheme's ability to provide enhanced financial security and improved quality of life for retirees, facilitated by the investment returns on the contributions made during their working years.

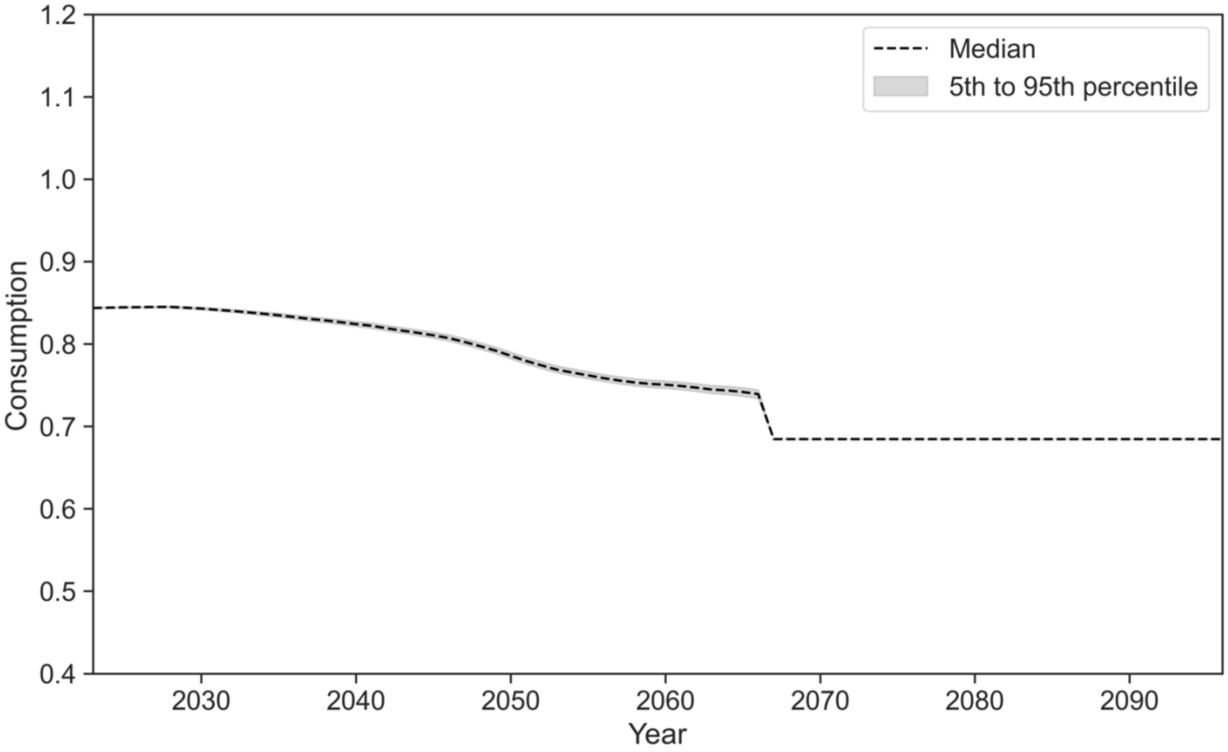
Figure 4.6. China - Consumption Profile within Optimal DC Scheme under Moderate Birth Rate Scenario



Data Source: Own Presentation

Figure 4.7 provides a detailed analysis of the consumption patterns within the DB scheme under China's moderate birth rate scenario. Unlike the Defined Contribution scheme, the DB scheme exhibits a contrasting trend where individuals experience a gradual decline in median consumption levels, moving from 0.85 down to 0.75 over the pre-retirement phase. This downward trajectory suggests that while the DB scheme offers stability and predictability in benefits, it may not keep pace with the rising cost of living or individual expectations over time. The figure also illustrates minimal variability in consumption, as represented by the narrow shadow area around the median consumption line. This reduced variability is a direct consequence of the low adjustment rates for fund imbalances within the DB scheme. Such adjustments are crucial in maintaining the scheme's stability but also limit its flexibility in responding to economic changes or shifts in the demographic landscape. Upon retirement, the consumption profile demonstrates a leveling effect, with median consumption stabilizing at approximately 0.68. This stabilization reflects the nature of the DB scheme, where retirees receive a predefined benefit that does not fluctuate significantly with market conditions or individual investment choices.

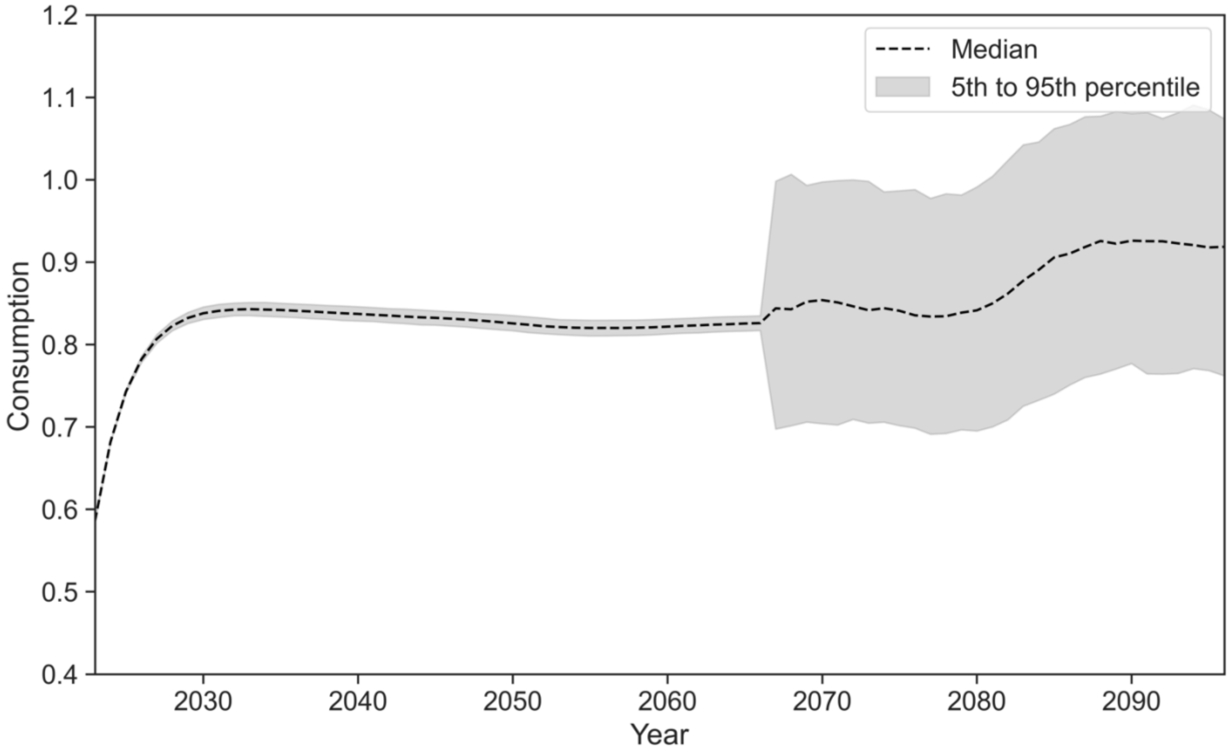
Figure 4.7. China - Consumption Profile within Optimal DB Scheme under Moderate Birth Rate Scenario



Data Source: Own Presentation

The exploration of the hybrid pension model, as depicted in Figure 4.8, under a moderate birth rate scenario in China, reveals a nuanced approach to managing consumption patterns across an individual's lifetime. Initially, there's an upward trajectory in median consumption levels before retirement, achieving stability around 0.83. This gradual increase signifies a balanced and well-managed growth in living standards as individuals approach retirement, facilitated by the hybrid model's design that combines the best features of both DC and DB schemes. Notably, the hybrid model exhibits a relatively narrow gap between the 5th and 95th percentiles, indicating lower variability in consumption outcomes for participants. This characteristic is attributed to the model's low adjustment rate for fund imbalances (α), which effectively minimizes the risk of wide disparities in retirement outcomes among individuals. Post-retirement, a slight increase in median consumption is initially observed, which then follows a steady rise, culminating in a median consumption of 0.90 by the maximum age considered in the study. This pattern contrasts with the more pronounced fluctuations and variability observed in the DC and DB schemes. The hybrid model's ability to ensure a stable and incrementally improving consumption level both before and after retirement underscores its effectiveness in providing a secure and enhanced quality of life for retirees.

Figure 4.8. China - Consumption Profile within Optimal Hybrid Scheme under Moderate Birth Rate Scenario

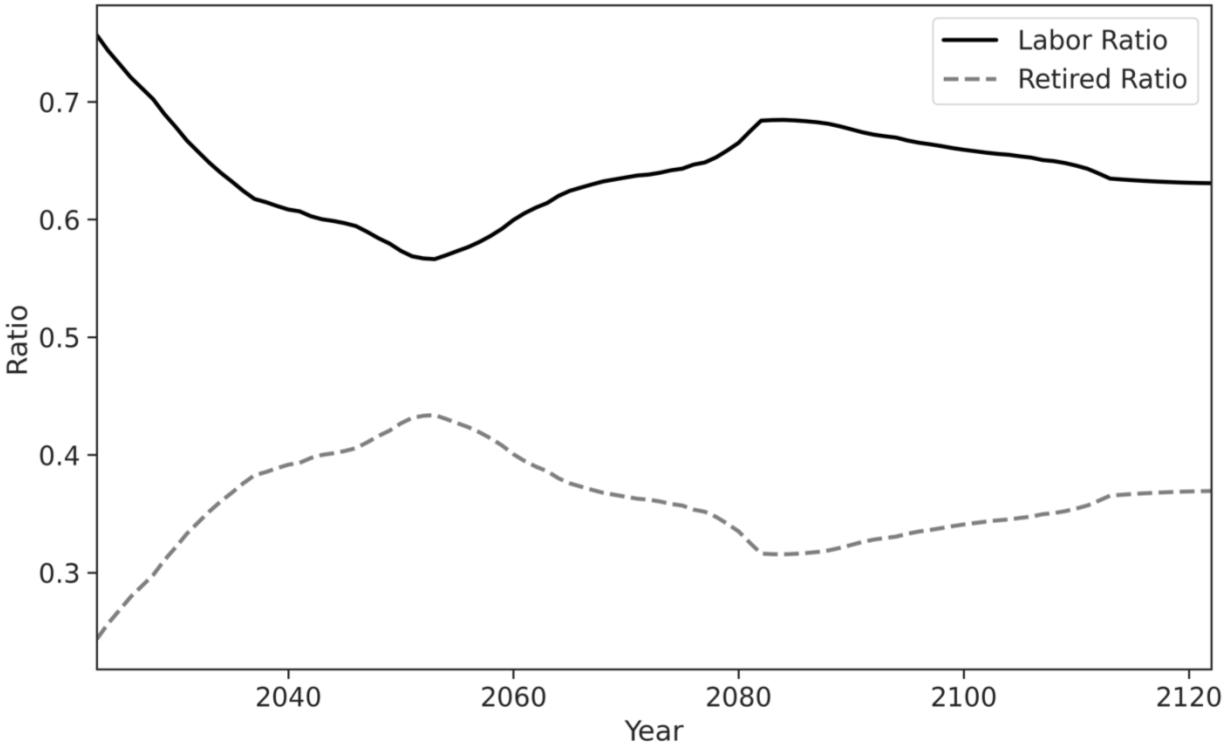


Data Source: Own Presentation

4.3.3. High Birth Rate

Under a high birth rate scenario, set at 14 births per 1,000 individuals, Figure 4.9 provides projections for the labor and retired ratios. Initially, the labor ratio is anticipated to decrease until around 2050, followed by a increase, reaching approximately 0.65 by 2080, before experiencing a slight downward adjustment to stabilize around 0.6 by 2120. In contrast, the retired ratio follows an opposite trajectory. Throughout the 100-year forecast period, the labor ratio consistently remains significantly higher than the retired ratio, with the gap between the two stabilizing at approximately 0.3 towards the end of the projection. This demographic configuration suggests stronger support for the pension system, with a broader base of contributors relative to beneficiaries, potentially easing the financial pressures on pension schemes and enhancing their sustainability over the long term.

Figure 4.9. China - Projections of the Labor Ratio and Retired Ratio over the Next 100 Years under Conditions of High Birth Rate



Data Source: Own Presentation

Table 4.3 displays the optimal parameters for DC, DB, and Hybrid pension schemes under high birth rate conditions. In this scenario, the DC scheme has a target contribution rate (p) of 0.1976, a significant adjustment rate for the replacement rate ($\beta = 0.7461$), and a moderate investment in risky assets ($\omega = 0.0417$), with a target benefit rate (b) of 1.2434, leading to a CEC of 0.8813. The DB scheme, with a target contribution rate of 0.0300 and an adjustment rate for contribution rate imbalances ($\alpha = 0.0208$), allocates a substantial portion to risky assets ($\omega = 0.3000$), with a target benefit rate of 0.7192, resulting in a CEC of 0.7652. The Hybrid model sets its contribution rate at 0.1188, with adjustment rates for fund imbalances ($\alpha = 0.0111, \beta = 0.9900$) and an investment in risky assets ($\omega = 0.0500$), with a target benefit rate of 0.9968, achieving the highest CEC of 0.9193 among the schemes.

When compared to Table 4.1 and 4.2 (low birth rate and moderate birth rate conditions), Table 4.3 shows varied adjustments in response to high birth rate conditions. CEC values in Table 4.3 are generally higher, suggesting that under high birth rate conditions, the pension schemes are optimized for enhanced welfare outcomes, as evidenced by the higher CEC values. The Hybrid scheme continues to offer the most favorable welfare outcomes, as indicated by its CEC, demonstrating its adaptability and effectiveness across different demographic scenarios.

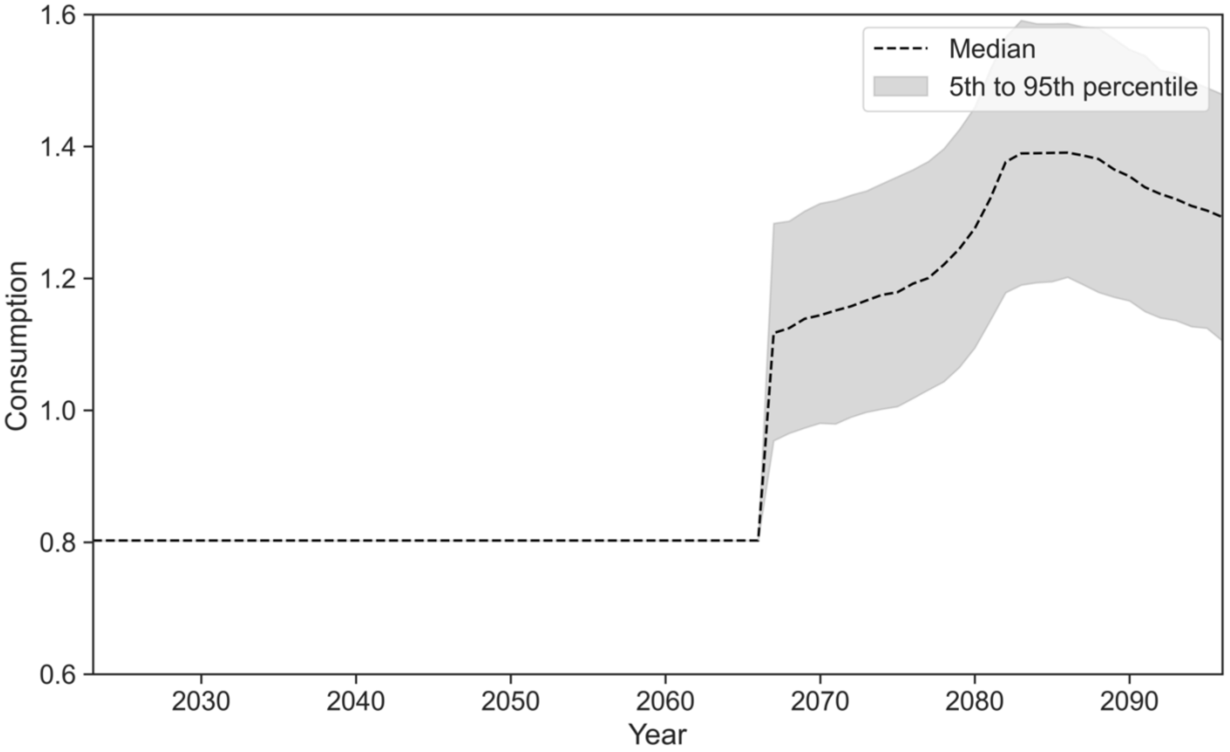
Table 4.3. China - Optimal Pension Schemes under Conditions of High Birth Rate

Pension Scheme	DC	DB	Hybrid
p	0.1976	0.0300	0.1188
α	-	0.0208	0.0111
β	0.7461	-	0.9900
ω	0.0417	0.3000	0.0500
b	1.2434	0.7192	0.9968
CEC	0.8813	0.7652	0.9193

Data Source: Own Presentation

In the high birth rate scenario presented in Figure 4.10, the DC scheme in China demonstrates a stable median consumption level of approximately 0.80 during the working years, providing a foundation of financial steadiness for individuals prior to retirement. Remarkably, as they transition into retirement, there's a significant uptick in median consumption, escalating to about 1.12, reflecting an enhanced standard of living afforded by the pension scheme. This upward trend in consumption continues, peaking at 1.4 by the year 2080, before experiencing a slight moderation to around 1.3 as individuals approach the maximum age considered in the study. This pattern underscores the DC scheme's capability to significantly improve retirees' welfare in a high birth rate context.

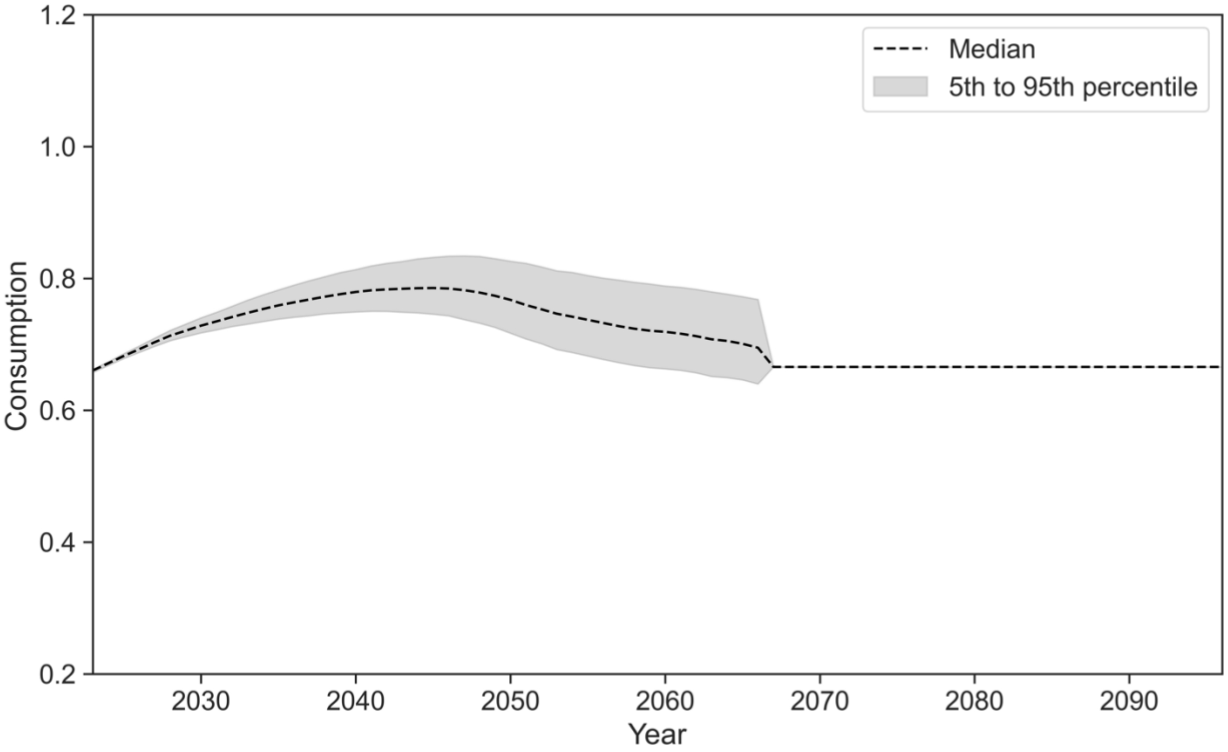
Figure 4.10. China-Consumption Profile within Optimal DC Scheme under High Birth Rate Scenario



Data Source: Own Presentation

Figure 4.11 illustrates the consumption dynamics within the Defined Benefit (DB) scheme in China, particularly under a high birth rate scenario. Before retirement, individuals' median consumption experiences fluctuations, oscillating between 0.65 and 0.80. This variation reflects an ongoing effort to balance immediate financial needs against the accumulation of sufficient retirement savings. However, a significant shift occurs upon retirement, where consumption levels notably stabilize at a median of approximately 0.75. This stabilization at a higher median consumption level, compared to the low and moderate birth rate scenarios, underscores the DB scheme's capacity to ensure a consistent and relatively higher standard of living for retirees in a high birth rate context.

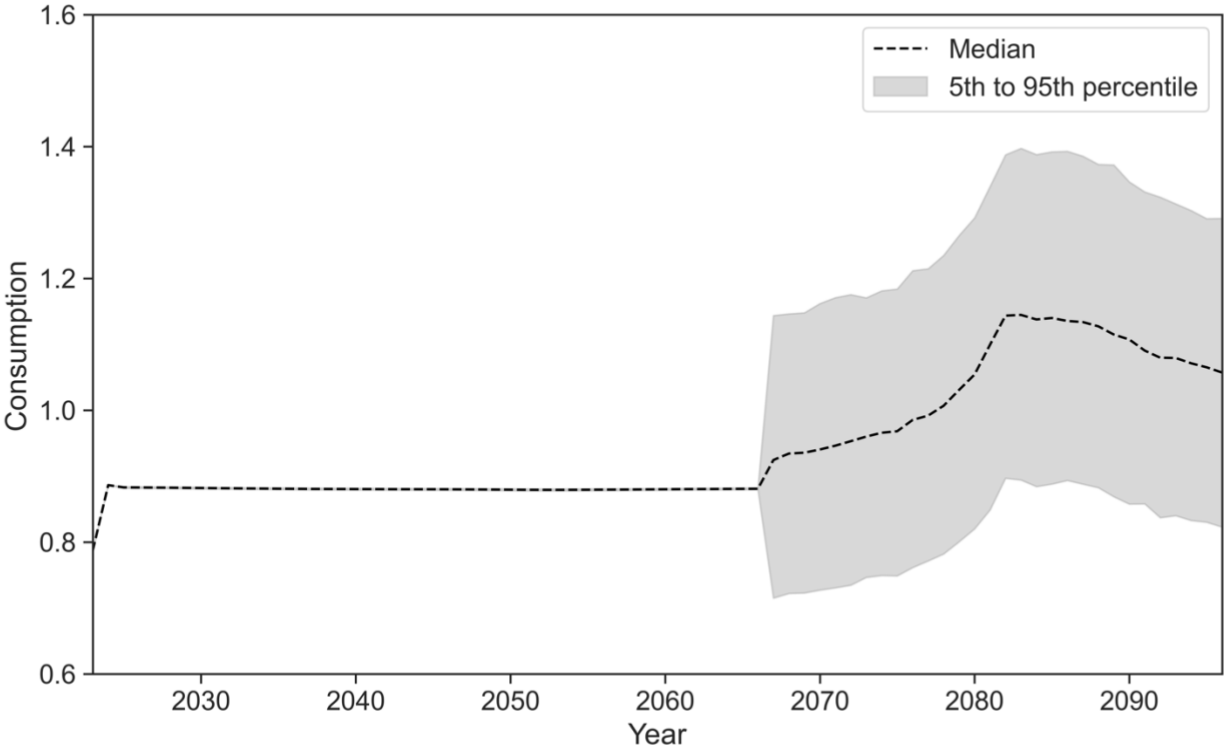
Figure 4.11. China - Consumption Profile within Optimal DB Scheme under High Birth Rate Scenario



Data Source: Own Presentation

The hybrid pension model, examined last, demonstrates a consistent median consumption level before retirement, stabilizing around 0.9. This model is characterized by a minimal disparity between the 5th and 95th percentiles, attributed to the low alpha value. Following retirement, there is an initial increase in median consumption, peaking at approximately 1.15 by 2080, before a gradual decline to a median level of 1.1 as individuals reach the maximum age considered. When compared to the DC scheme, the hybrid model offers higher pre-retirement consumption levels and exhibits a more stable but lower pattern of consumption post-retirement. Relative to the DB scheme, the hybrid model provides consistently higher consumption levels both before and after retirement.

Figure 4.12. China - Consumption Profile within Optimal Hybrid Scheme under High Birth Rate Scenario



Data Source: Own Presentation

4.4. Robustness Checks

In order to strengthen the reliability and applicability of our results and conclusions, we conduct a series of robustness checks. These investigations extend to different aspects of pension scheme design, assessing their impact on the model results in a high-risk investment environment. In addition, we examined the impact of delaying the retirement age on the viability and performance of different pension schemes.

4.4.1. Adjustment of Investment Strategies in Pension Schemes

By setting the investment parameter omega (ω) to a fixed value of 0.3, the allocation towards risky assets in pension funds reaches its maximum permissible limit, necessitating a modification of the original optimal parameters from $\{p, \alpha, \beta, \omega\}$ to $\{p, \alpha, \beta\}$. This adjustment is key to analyzing how the DC, DB, and Hybrid pension schemes perform under a heightened risk asset allocation strategy. According to Table 4.4, with a retirement age set at 60, the CEC values across different birth rate scenarios indicate that the Hybrid scheme outperforms the others, yielding the highest CEC values: 0.7090 under a low birth rate, 0.7963 under a moderate birth rate, and 0.8027 under a high birth rate scenario. This consistently superior performance of the Hybrid scheme, as indicated by these specific numbers, demonstrates its effectiveness and potential for providing the most favorable welfare outcomes for participants under the given investment strategy.

Table 4.4. China - Adjusted Optimal Parameters for Pension Schemes in a High-Risk Investment Environment

Pension Scheme	DC	DB	Hybrid
Low birth rate	0.5971	0.6756	0.7090
Moderate birth rate	0.6755	0.7345	0.7963
High Birth Rate	0.7332	0.7532	0.8027

Data Source: Own Presentation

4.4.2. Delay Retirement

The issue of an aging population has led countries like China to consider policies for delaying retirement as a matter of urgency, with the goal of ensuring the sustainability of pension systems (Wu et al., 2022; Zhang et al., 2023). Research by Deng et al. (2023) indicates that increasing the retirement age from 60 to 65 could significantly bolster the finances of China's pension system. The China Pension Report 2023, published by the Social Security Laboratory at the Chinese Academy of Social Sciences on December 29, 2023, highlights the pressing need to implement a delayed retirement policy and suggests that the revised statutory retirement age could be set at 65 (Du, 2024). In light of this, our model calibration also examines the stability of pension schemes under a reform scenario where the retirement age is increased from 60 to 65.

Table 4.5 reflects the CEC values for DC, DB, and Hybrid pension schemes, assuming a retirement age of 65 across various birth rate scenarios. For the low birth rate scenario, the CEC values are 0.7130 for DC, 0.7516 for DB, and 0.8077 for the Hybrid scheme. In the moderate birth rate scenario, the CEC increases to 0.8601 for DC, 0.8312 for DB, and 0.9087 for the Hybrid scheme. Under the high birth rate conditions, the CEC values are the highest, at 0.8910 for DC, 0.8383 for DB, and 0.9658 for the Hybrid scheme. Overall, the Hybrid pension scheme consistently offers the highest welfare across all birth rate scenarios when the retirement age is set at 65.

Comparing these results with those from a retirement age of 60, it's apparent that delaying retirement to 65 improves the CEC for all pension schemes and across all birth rate scenarios. The increase in CEC indicates that individuals would experience a higher level of equivalent consumption and, by extension, welfare, with a delayed retirement.

Table 4.5. China - Optimal Pension Schemes CEC with Retirement Age of 65

Pension Scheme	DC	DB	Hybrid
Low birth rate	0.7130	0.7516	0.8077
Moderate birth rate	0.8601	0.8312	0.9087
High Birth Rate	0.8910	0.8383	0.9658

Data Source: Own Presentation

4.5. Discussion and Conclusion

This research conducts a comprehensive examination of a proposed collective hybrid pension scheme, positioning it in contrast to traditional Defined Contribution (DC) and Defined Benefit (DB) pension schemes. Our analysis is rooted in the Overlapping Generations (OLG) framework pioneered by Heer and Maußner (2009), which serves as the bedrock of our population model. This model, further enhanced by the incorporation of a survival model and a demographic model, effectively calculates the probability that a member will survive at each age, thereby accurately measuring benefits and assessed contributions based on the actual demographic distribution of each generation. The model is deployed in conjunction with Monte Carlo simulations and infused with critical parameters obtained from existing academic literature, enriching the overall scope of our study. To infuse our simulation model with realism and specificity, we meticulously calibrated it using data that accurately mirrors China's demographic conditions. These methodological choices, combined with the execution of 1000 simulations and a series of robustness checks, rendered our findings both robust and reliable.

In Chapter 4's analysis, the Defined Contribution (DC), Defined Benefit (DB), and Hybrid pension schemes are scrutinized under varying demographic conditions in China, with particular attention to their performance through Certainty Equivalent Consumption (CEC) values. The DC scheme, characterized by individual contributions and investments, typically provides a stable yet less adaptive response to demographic shifts, as reflected in its CEC values across different birth rate scenarios. The DB scheme, relying on predefined benefits, shows a level of stability and predictability in its outcomes, but its rigidity may limit responsiveness to changing economic conditions. The Hybrid scheme, however, stands out for its flexibility and resilience, combining the best features of DC and DB models. It consistently offers the highest CEC values, indicating superior welfare outcomes for participants. This is attributable to its dynamic structure, which can adjust to demographic and economic fluctuations more effectively, thereby providing a more sustainable and participant-friendly approach within China's pension landscape.

The robustness checks conducted in Chapter 4 serve to validate the reliability and applicability of the pension models under various conditions, particularly within the high-risk investment environment and scenarios involving delayed retirement ages. These checks reveal that the Hybrid pension scheme maintains its superiority in adaptability and performance, even under stress tests that simulate extreme market conditions and policy changes. Notably, when investment in risky assets is maximized , the Hybrid scheme continues to outperform the DC and DB schemes, showcasing its robust design that effectively balances risk and reward. Furthermore, increasing the retirement age from 60 to 65 enhances the CEC across all schemes, with the Hybrid model again demonstrating the most significant improvement in welfare outcomes. Moreover, extending the retirement age from 60 to 65 improves the CEC for all schemes, with the Hybrid model showing the greatest welfare enhancement. This finding corroborates Deng et al., (2023), suggesting that postponing retirement to 65 could be a viable strategy for Chinese policymakers to boost citizens' welfare, with the Hybrid scheme optimally positioned to capitalize on such policy adjustments.

As a crucial addition to the body of existing research, the findings of this study resonate harmoniously with prior scholarly work. The empirical evidence garnered through our research echoes the premises put forward by Chen et al. (2023) and Chen et al. (2022), further highlighting the practical advantages of intergenerational risk-sharing in hybrid pension systems. Moreover, our investigation of the adaptability of the hybrid scheme, particularly in dealing with demographic transformations and increased life expectancy, is in line with the insights provided by Romp and Beetsma (2020) as well as Rong et al. (2023). This convergence of findings across diverse studies lends significant credibility to our conclusions, thereby solidifying the theoretical underpinnings for the adoption of hybrid pension schemes in meeting the challenges inherent to conventional pension systems.

In conclusion, this study makes a substantial contribution to the field of pension scheme design by integrating demographic struture into our model, a step that brings a higher level of accuracy and practical relevance to our findings. The intergenerational risk-sharing approach, a cornerstone of the proposed hybrid pension scheme, is brought into sharp focus, illuminating its pivotal role in managing demographic risks and enhancing overall welfare. The hybrid pension scheme, which we meticulously propose and analyze, provides a compelling avenue toward fortifying the sustainability of pension systems in the face of a rapidly aging global population. Importantly, our model's integration of real-world mortality rates allows for a more precise evaluation of the system's response to demographic shifts, lending further credence to our results. These findings present valuable insights for policyholders, suggesting that a transition towards hybrid pension schemes could provide a viable solution to the challenges currently faced by traditional pension systems. The results could potentially guide and drive pension reform, fostering policies that ensure stability and long-term welfare for all generations. In particular, China - which faced a negative population growth rate of -0.06% in 2022 - could benefit from our study's insights. The transition towards hybrid pension schemes could significantly mitigate demographic risks and ensure sustained welfare across generations in China's unique demographic context. We hope that our study will serve as a reference for future research and policy development, particularly for countries like China experiencing significant demographic shifts and challenges.

Chapter 5. Optimal Investment Strategies-Poland Simulation

5.1. Introduction

In Chapter 5, we shift focus from China to explore the pension system landscape in Poland, building on the foundational model calibrated in Chapter 4. This examination is aimed at adapting and rigorously testing our model within the unique confines of Poland's socio-economic environment, which is characterized by distinct pension scheme attributes and demographic nuances. Notably, Poland shares with China the implementation of a Defined Contribution (DC) pension scheme as a strategic response to looming demographic challenges. Both countries are on a trajectory of proactive pension reforms aimed at bolstering the sustainability and adequacy of their pension systems amidst aging populations and shifting economic tides.

However, it is crucial to acknowledge the differences between the two countries, especially regarding specific pension scheme policies. These differences include constraints on investment strategies, variations in working and retirement ages, and differing employer contribution rates. Such variations play a critical role in shaping each country's pension system's effectiveness and sustainability.

Furthermore, the demographic landscape in Poland presents a stark contrast to that of China, as elucidated in the findings from Chapter 2. The comparative analysis of the current and projected population structures reveals divergent trends between the working and retired populations in both countries. Such demographic discrepancies are anticipated to significantly influence the operational dynamics and strategic needs of their respective pension systems. This chapter delves into these differences and their implications, setting the stage for a comprehensive analysis of Poland's pension system within a global context of shifting demographic patterns and pension reform imperatives.

5.2. Assumption

Age of Entry (E)

According to Gov.pl (n.d.-b), the standard definition of an employee encompasses natural persons who have reached the age of 18. However, in special circumstances, individuals as young as 15 may be employed, particularly for roles such as apprenticeships or light work. Furthermore, the eligibility for receiving a social pension begins at 18 years of age (Gov.pl, n.d.-a). Thus, for the context of this analysis, the age of entry into the workforce is set at 18.

Age of retirement (R)

In Poland, the official retirement age has been standardized to 60 years for women and 65 years for men since 1 October 2017, as per ZUS (2023). Upon reaching the stipulated retirement age, individuals have the discretion to either continue working or opt for retirement. For the purpose of this study, which considers the population in aggregate without distinguishing by gender, the retirement age is uniformly set at 65.

Weight investment in risky asset (w)

According to Ministerstwo Rodziny Pracy i Polityki Społecznej (n.d.), AXA OFE's investment policy assumes that the average share of equities in the Fund's assets will be 80%, but deviation from this level is allowed depending on the expected economic situation. Therefore, the maximum weight invest in risky asset of the pension fund is set to be 80%.

Maximum Age (M)

Building on the analysis presented in Chapter 2, the purposes of this simulation, we have established the maximum age parameter at 90.

Employer Contribution

According to the European Commission (2019) report on Employee Capital Plans (ECPs) in Poland, funding for ECPs comes from a combination of employer, employee, and state budget contributions. The mandatory employer contribution is set at 1.5% of an employee's wage, with an option for employers to make an additional voluntary contribution of up to 2.5% of the wage. For the purposes of this analysis, the total employer contribution is assumed to be 4%, encompassing both the mandatory and maximum voluntary contributions.

5.3. Numerical Results

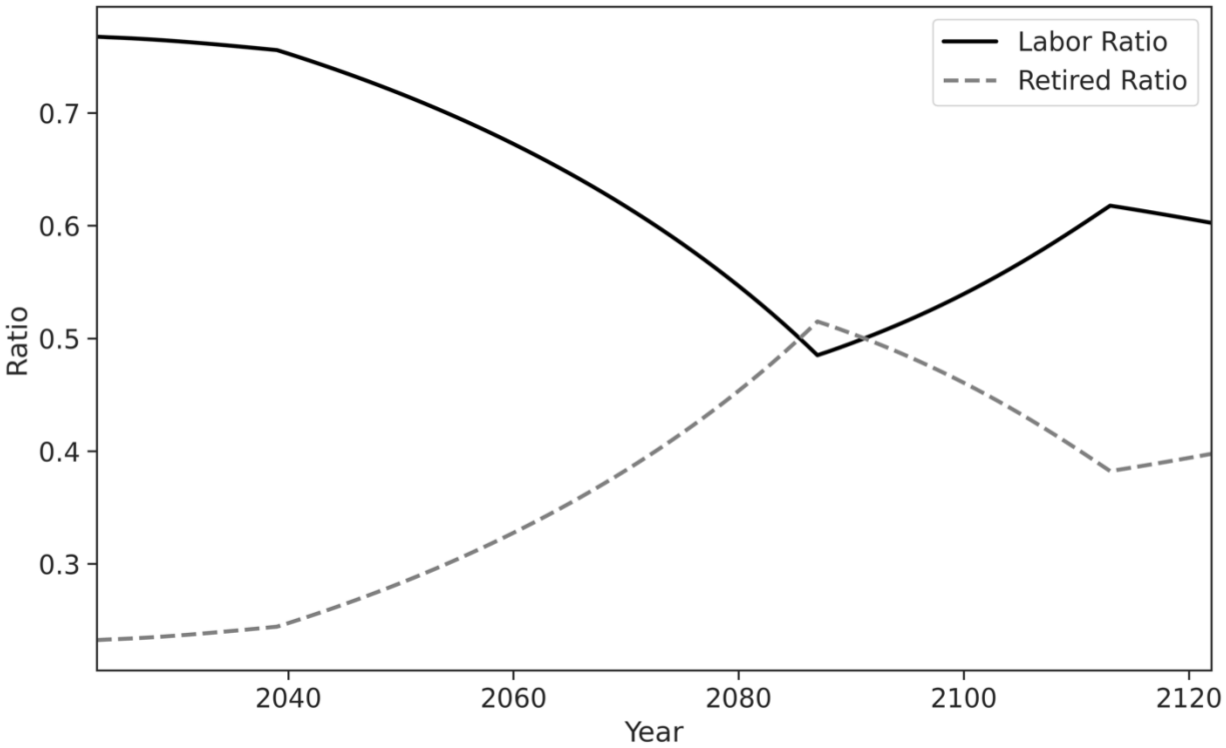
Mirroring the assumptions established in Chapter 4, the financial market framework for the Poland simulation incorporates both a risk-free and a risky asset. The risk-free asset is characterized by a non-stochastic interest rate (r) set at 2%. Conversely, the risky asset's price dynamics adhere to a geometric Brownian motion with a constant drift (μ) of 0.06 and a volatility (σ) of stock returns pegged at 15%. The model also integrates a risk aversion coefficient (γ) of 5. All pension schemes are initiated on a fully funded basis, ensuring assets (A_0) equal liabilities (L) at the outset.

Furthermore, the two-period Overlapping Generations (OLG) model is applied to dissect the population into working and retirement phases. The working phase spans from ages 18 to 65, while the retirement phase covers ages 66 to 90. In alignment with the methodology of Chapter 4, the calibration of the pension model for Poland includes scenarios based on varying birth rates. These rates are categorized into three distinct levels: low (6 per 1,000), moderate (10 per 1,000), and high (14 per 1,000). These categories reflect historical birth rate trends observed in Poland from 1997 to 2022, as detailed in Chapter 2, providing a realistic framework for assessing potential demographic impacts on pension outcomes.

5.3.1. Low Birth Rate

Figure 5.1 projects the future proportions of the working and retired populations over a century, within a low birth rate scenario of 6 per 1000 people. The labor ratio, indicative of the working population's share, is expected to decline, hitting its lowest around 2080, and subsequently recovering to approximately 0.6. Meanwhile, the retirement ratio, reflecting the share of the retired population, follows an increasing trend until it peaks and then decreases after 2090. When juxtaposed with similar projections for China, the labor and retired ratios for Poland demonstrate less volatility over time. The labor ratio in Poland displays a smoother transition into its eventual recovery phase.

Figure 5.1. Poland - Projections of the Labor Ratio and Retired Ratio over the Next 100 Years under Conditions of Low Birth Rate



Data Source: Own Presentation

Table 5.1 presents the optimal parameters for DC, DB, and Hybrid pension schemes in Poland, specifically under a low birth rate scenario. The table details each scheme's target contribution rate (p), the speed of adjustment for contribution rate imbalances (α), the speed of adjustment for replacement rate imbalances (β), the investment in risky assets (ω), the target benefit rate (b), and the resulting CEC. For the DC scheme, we see a relatively target high contribution rate ($p = 0.4089$) paired with a moderate adjustment rate for replacement rate imbalances ($\beta = 0.3911$) and a conservative investment in risky assets ($\omega = 0.0282$), resulting in a CEC of 0.6393. This suggests that participants under the DC scheme in a low birth rate environment might experience lower welfare outcomes compared to the other schemes. The DB scheme, with a target contribution rate of 0.1041 and an adjustment rate for contribution rate imbalances (α = 0.0202), directs a high proportion of its funds into risky assets ($\omega = 0.7860$), indicative of a more aggressive investment strategy. Its target benefit rate ($b = 0.5712$) leads to a CEC of 0.6909, reflecting a moderate welfare level for its participants. Lastly, the Hybrid model, which combines features of both DC and DB schemes, shows a balanced contribution rate ($p = 0.1566$), a higher adjustment rate for contribution rate imbalances ($\alpha = 0.0310$), a relatively conservative adjustment for replacement rate imbalances ($\beta = 0.0101$), and a substantial investment in risky assets ($\omega = 0.4421$). With a target benefit rate of 0.7793, the Hybrid scheme achieves the highest CEC of 0.7970, suggesting it provides the best welfare outcomes among the three schemes.

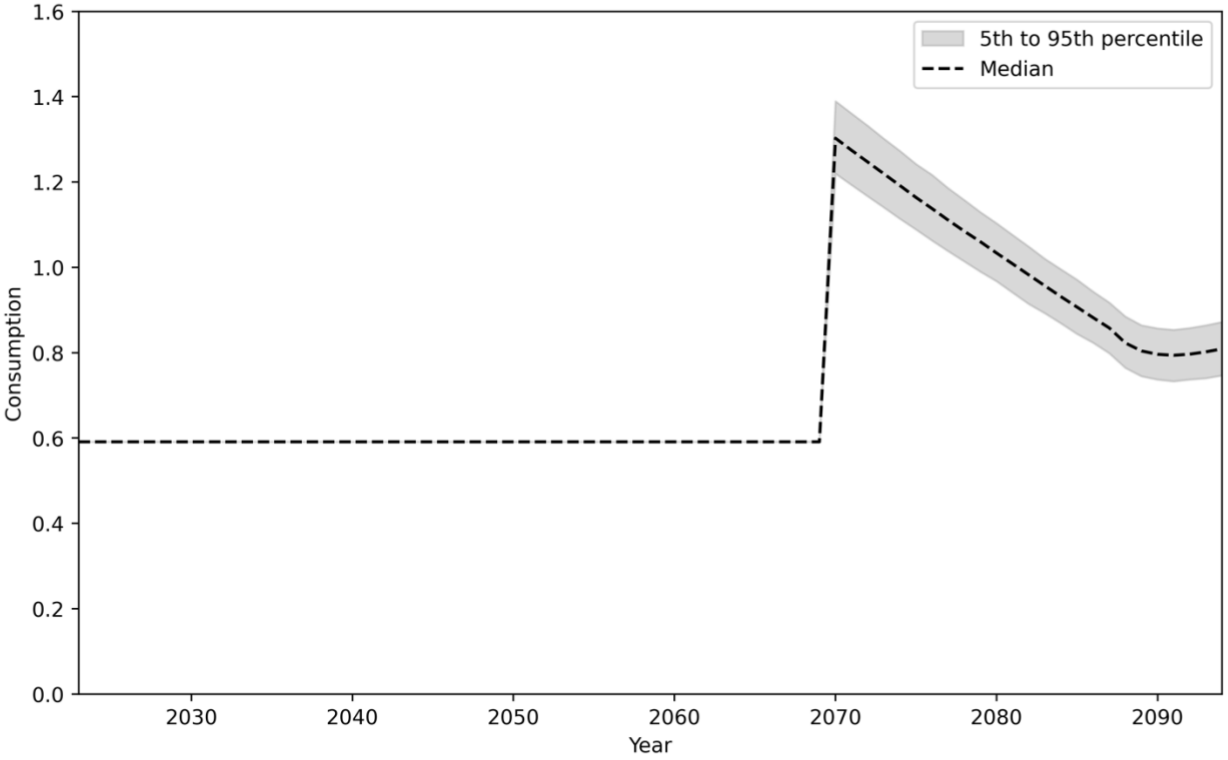
Table 5.1. Poland - Optimal Pension Schemes under Conditions of Low Birth Rate

Pension Scheme	DC	DB	Hybrid
p	0.4089	0.1041	0.1566
α	-	0.0202	0.0310
β	0.3911	-	0.0101
ω	0.0282	0.7860	0.4421
b	1.7797	0.5712	0.7793
CEC	0.6393	0.6909	0.7970

Data Source: Own Presentation

Using 5,000 Monte Carlo simulations, the study scrutinises the consumption patterns of an individual starting his/her career in 2022 and follows him/her throughout his/her lifecycle from age 18 to age 90. Figure 5.2 outlines the consumption profile within the Optimal DC Scheme in Poland under a low birth rate scenario. From the pre-retirement period (year 2023 to 2069), the data reflects remarkable stability in the consumption levels for individuals within this pension scheme, with 5% quantile, median and 95% quantile consumption remaining consistent at 0.59. However, a dramatic shift occurs post-2070 as individuals transition into retirement. The median consumption level increases significantly, rising from 1.2 in 2070 to a peak in the same year, which deviates from the stable pre-retirement pattern observed and then decrease to around 0.8 at the end.

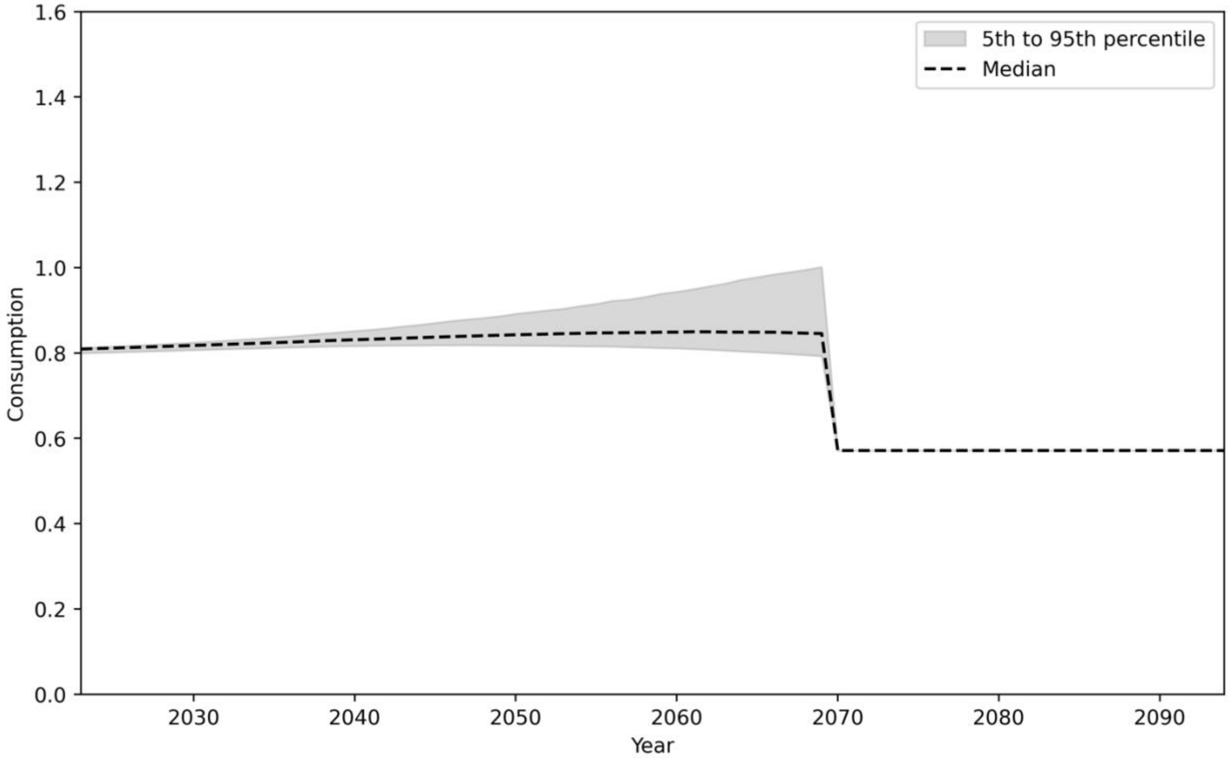
Figure 5.2. Poland - Consumption Profile within Optimal DC Scheme under Low Birth Rate Scenario



Data Source: Own Presentation

Figure 5.3 illustrates the consumption patterns within the Defined Benefit (DB) pension scheme in Poland, under a scenario of low birth rates. The data reveals that from the beginning of the working age in 2023, consumption levels—across the 5th, 50th (median), and 95th percentiles—start at approximately 0.80 and exhibit a gradual increase over time. After 2061, the median consumption begins to show a slight decline. However, the transition to retirement marks a significant alteration in consumption patterns. Starting in 2070, consumption levels standardize across all quantiles at 0.571, a notable decrease from pre-retirement levels. This sudden change reflects the DB scheme's nature, where retirement benefits are typically defined and do not fluctuate with market performance or individual contributions as with the DC scheme. Consequently, the post-retirement phase under the DB scheme sees uniform and stable consumption, which could be indicative of a strong safety net provided by the DB plan but also signals a potential reduction in consumption ability compared to the working years. The consistent post-retirement consumption level suggests that while there may be less variability and risk for retirees, there is also a fixed ceiling on potential consumption benefits.

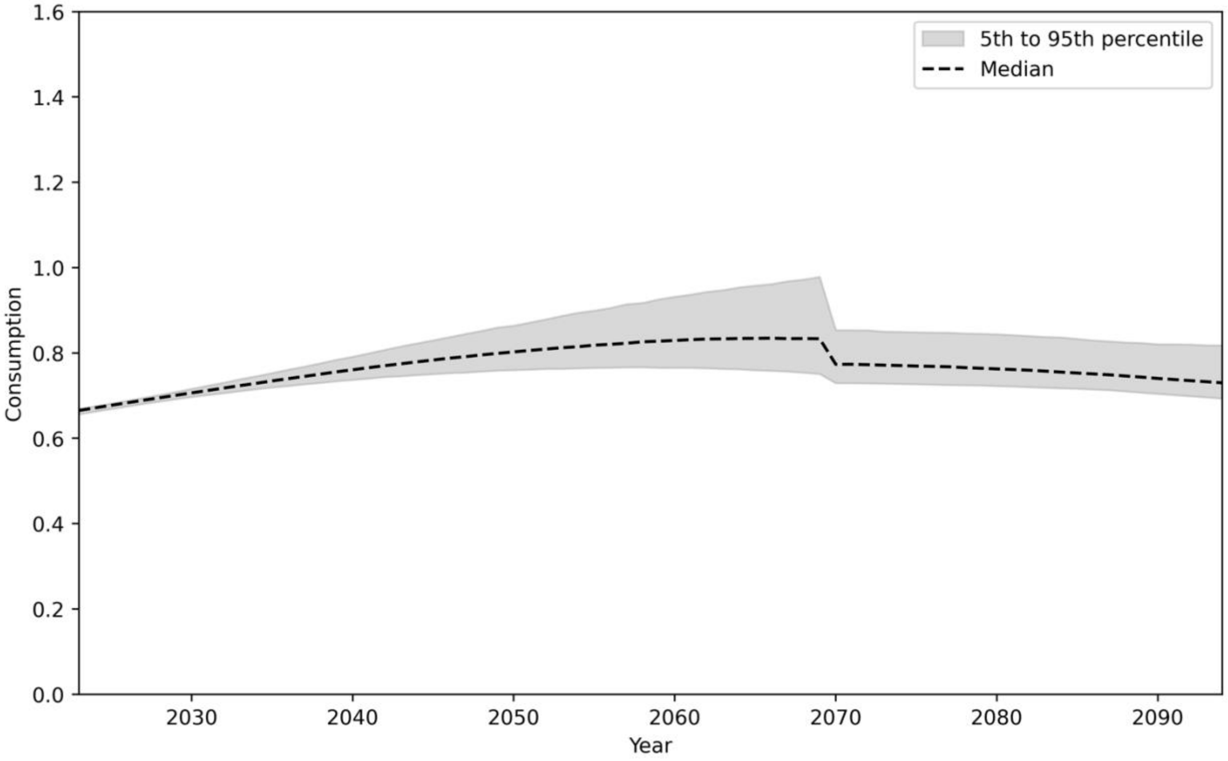
Figure 5.3. Poland - Consumption Profile within Optimal DB Scheme under Low Birth Rate Scenario



Data Source: Own Presentation

Figure 5.4 presents the consumption trends under the Optimal Hybrid Pension Scheme in Poland under a low birth rate context. This scheme, designed to incorporate elements of both DC and DB plans, illustrates a unique consumption trajectory from 2023 through to 2069. Initially, consumption levels start at around 0.66, with both the 5% and 95% quantiles, as well as the median (50% quantile), evidencing a consistent upward trend over the working years, culminating in a median consumption of approximately 0.83 by 2069. Post-retirement, unlike the sharp changes in the DB plan, the hybrid plan is more moderate, with consumption levels adjusting slightly after 2070 but mainly stabilizing at around 0.77, demonstrating the ability of the hybrid plan to provide a stable income in retirement, in line with its objective of combining the predictability of the DB plan with the growth potential of the DC plan.

Figure 5.4. Poland - Consumption Profile within Optimal Hybrid Scheme under Low Birth Rate Scenario

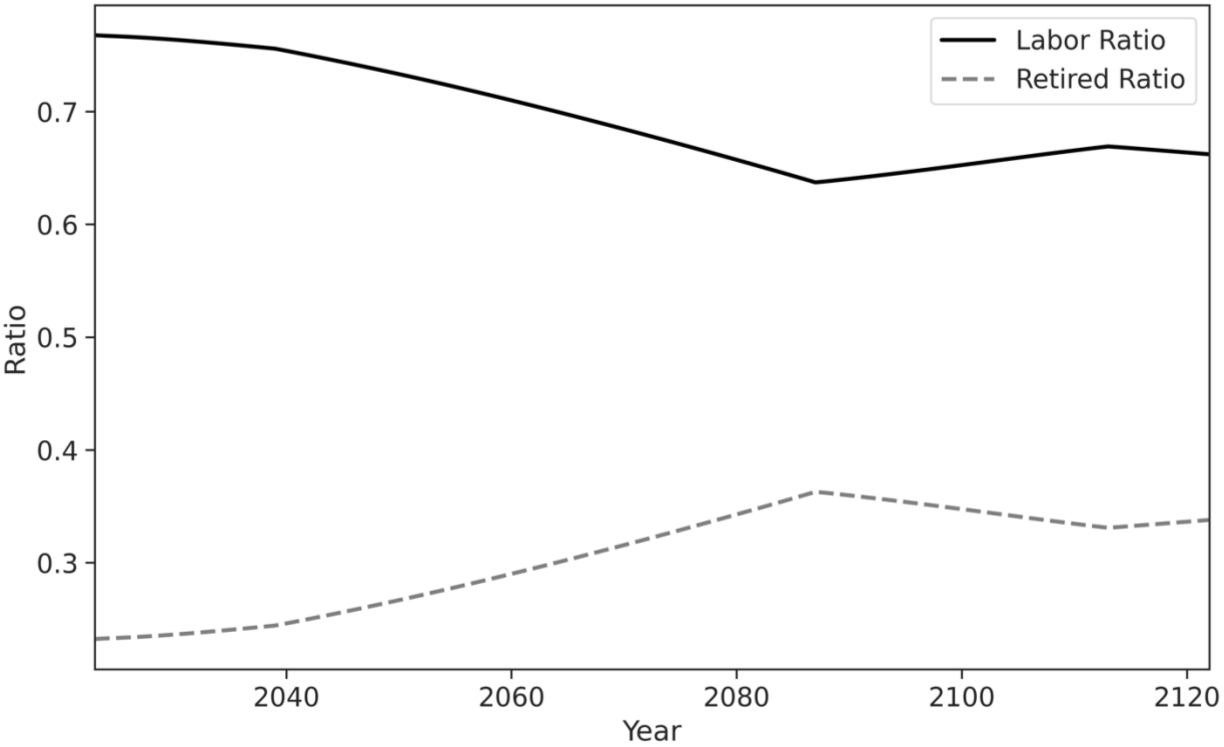


Data Source: Own Presentation

5.3.2. Moderate Birth Rate

Figure 5.5 presents the future projections of labor and retirement rates in Poland over the next century, considering a scenario with a moderate birth rate. Throughout the entire period, the labor rate consistently surpasses the retirement rate, illustrating a demographic structure where the working-age population remains dominant over the retired cohort. From the starting point in 2023, there's a gradual decline in the labor rate, reaching a low of 0.65 by 2090. After this point, there's an observable uptick, leading to a stabilization of the labor rate. This trend contrasts with the low birth rate scenario, where the retirement rate significantly narrows the gap with the labor rate. In the moderate birth rate scenario, the proportion of retirees remains considerably lower than that of the working population, indicating a less pronounced aging effect on the demographic composition.

Figure 5.5. Poland - Projections of the Labor Ratio and Retired Ratio over the Next 100 Years under Conditions of Moderate Birth Rate



Data Source: Own Presentation

Table 5.2 outlines the optimal parameters for DC, DB, and Hybrid pension schemes in Poland, under moderate birth rate conditions. The DC scheme, aiming for a relatively high target contribution rate (p) of 0.3136, adopts a conservative strategy towards investment in risky assets, with only 1.69% (ω) of assets allocated here, and sets an ambitious target benefit rate (b) at 1.4019. Its CEC value stands at 0.7447. In contrast, the DB scheme specifies a lower target contribution rate of 0.1678, a higher allocation to risky assets ($\omega = 0.3361$), and an adjustment rate for contribution rate imbalances ($\alpha = 0.0378$), aiming for a target benefit rate of 0.8240 and achieving a CEC of 0.7508. The Hybrid model combines features of both, setting its target contribution rate at 0.2092, with fund imbalance adjustment rates ($\alpha = 0.0136, \beta = 0.0165$), a significant proportion of investments in risky assets ($\omega = 0.6271$), and a target benefit rate of 0.9881, resulting in the highest CEC of 0.8073. This comparison indicates the Hybrid scheme's superior welfare outcome for participants, with its balanced approach and efficient risk management, making it still the optimal choice under moderate birth rate conditions.

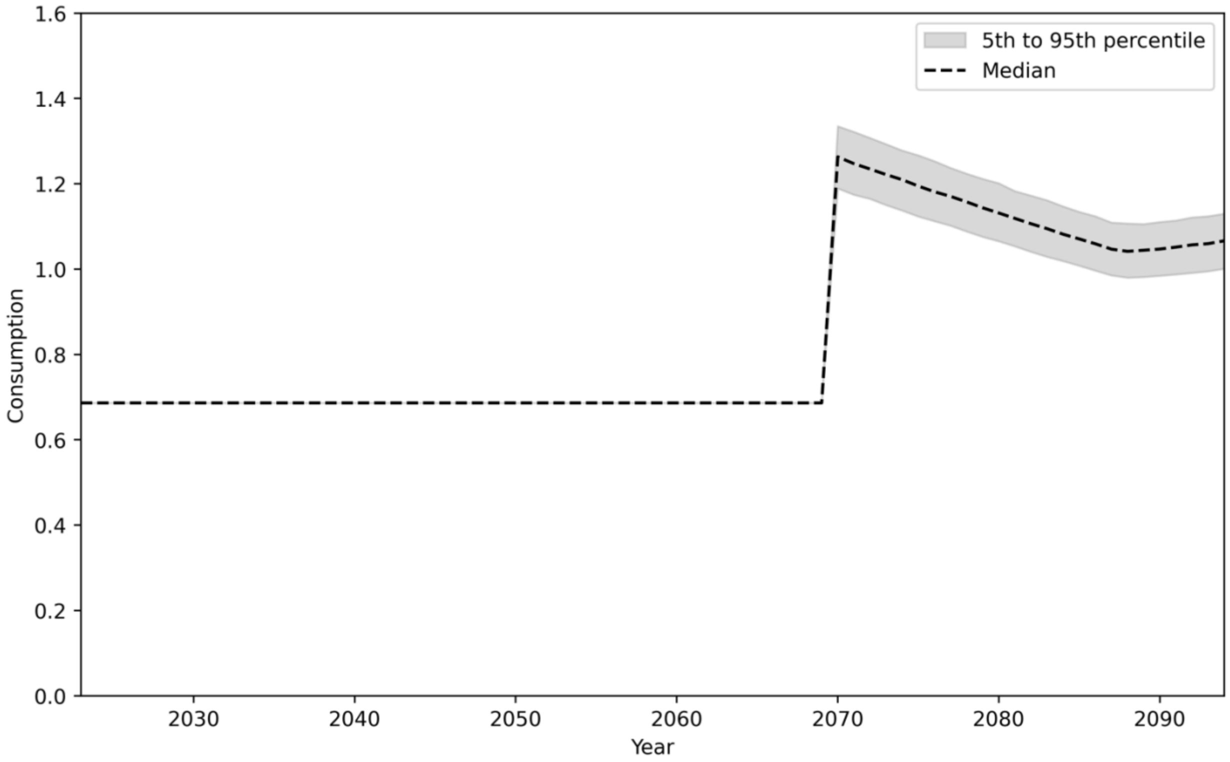
Table 5.2. Poland - Optimal Pension Schemes under Conditions of Moderate Birth Rate

Pension Scheme	DC	DB	Hybrid
p	0.3136	0.1678	0.2092
α	-	0.0378	0.0136
β	0.7119	-	0.0165
ω	0.0169	0.3361	0.6271
b	1.4019	0.8240	0.9881
CEC	0.7447	0.7508	0.8073

Data Source: Own Presentation

Figure 5.6 illustrates a fascinating shift in the consumption patterns of individuals within the optimal DC pension scheme in Poland under a moderate birth rate scenario, covering the years 2023 to 2094. Initially, from 2023 until just before 2070, the consumption levels are strikingly consistent, with all quantiles reflecting a uniform value of approximately 0.69. However, in 2070, there's a significant inflection point where consumption levels abruptly rise, indicating the commencement of retirement. The median consumption spikes to 1.26, showcasing a substantial increase in expected consumption at retirement onset. Following this peak, there's a slight decrease and then a stabilization of consumption levels around 1.04, illustrating the post-retirement consumption trend which, despite the initial post-retirement drop, remains significantly higher than the pre-retirement levels.

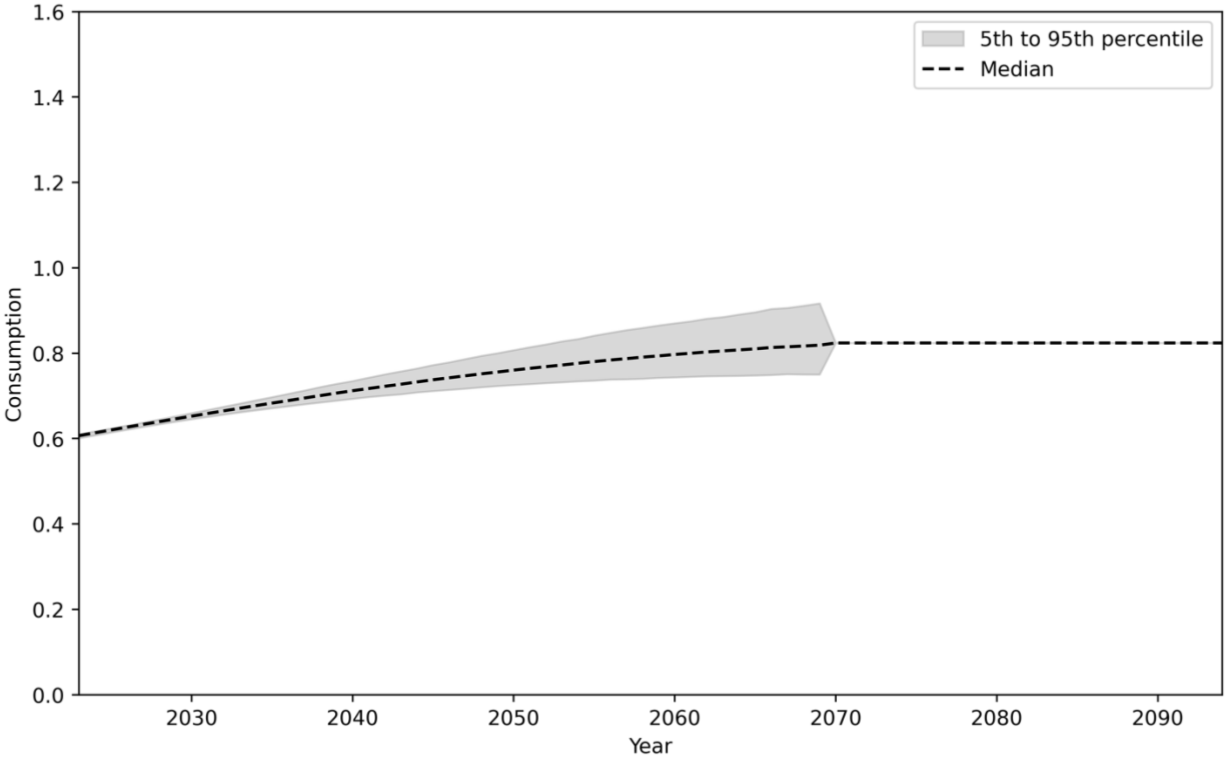
Figure 5.6. Poland - Consumption Profile within Optimal DC Scheme under Moderate Birth Rate Scenario



Data Source: Own Presentation

Figure 5.7 shows the consumption profile of the optimal DB plan. Initially, the profile shows a gradual increase in consumption across all quantiles from 2023 onwards, with a steady year-on-year increase starting at a median consumption of 0.60 in 2023. This trend reflects an increasing pattern of consumption as the individual ages, with the median consumption reaching approximately 0.82 by 2069. At retirement, all quantiles shift uniformly to a fixed consumption level of 0.82. This stability is a feature of the DB pension scheme, which is designed to provide retirees with a steady stream of post-retirement income, ensuring a secure and predictable financial environment for the rest of their lives.

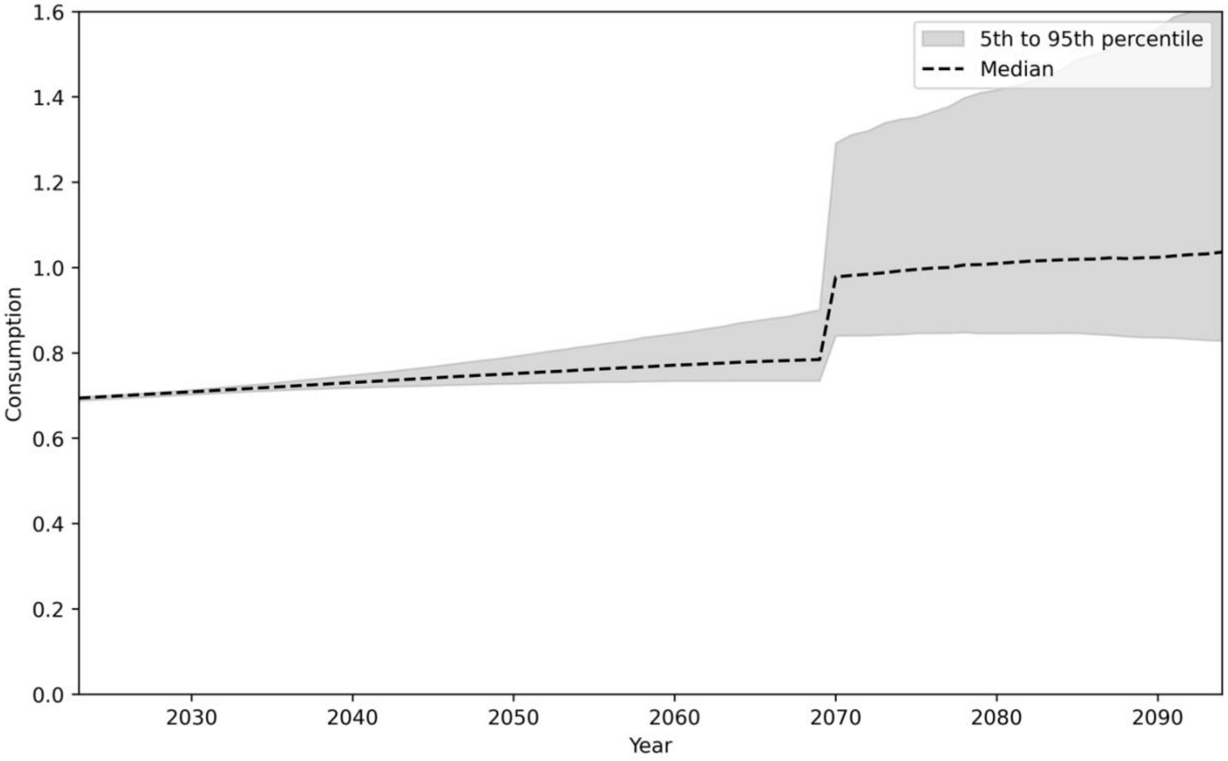
Figure 5.7. Poland - Consumption Profile within Optimal DB Scheme under Moderate Birth Rate Scenario



Data Source: Own Presentation

The consumption profile depicted in Figure 5.8 for the optimal Hybrid Scheme in Poland provides insightful trends into the expected financial well-being of individuals under this pension plan. The period from 2023 to 2069 is characterized by a gradual and consistent increase in median consumption, gently ascending from approximately 0.70 to 0.79. A notable shift occurs in 2070, where a significant leap in consumption levels is observed. The median consumption surges from around 0.79 to 0.92, marking a distinct increase in the standard of living afforded by the pension benefits. This upward trajectory continues through to 2094, where the median consumption reaches approximately 1.03, indicating sustained improvement in retirees' financial status over the observed period. The widening spread between the 5% and 95% quantiles post-2070 underscores the influence of the Hybrid Scheme's investment strategy, particularly its allocation to risky assets. The larger spread suggests greater variability in outcomes, reflecting the inherent risk and potential reward from investing in volatile assets. This aspect of the scheme's design highlights the balance between risk and return, aiming to maximize long-term benefits for retirees while managing the uncertainties associated with investment markets.

Figure 5.8. Poland - Consumption Profile within Optimal Hybrid Scheme under Moderate Birth Rate Scenario

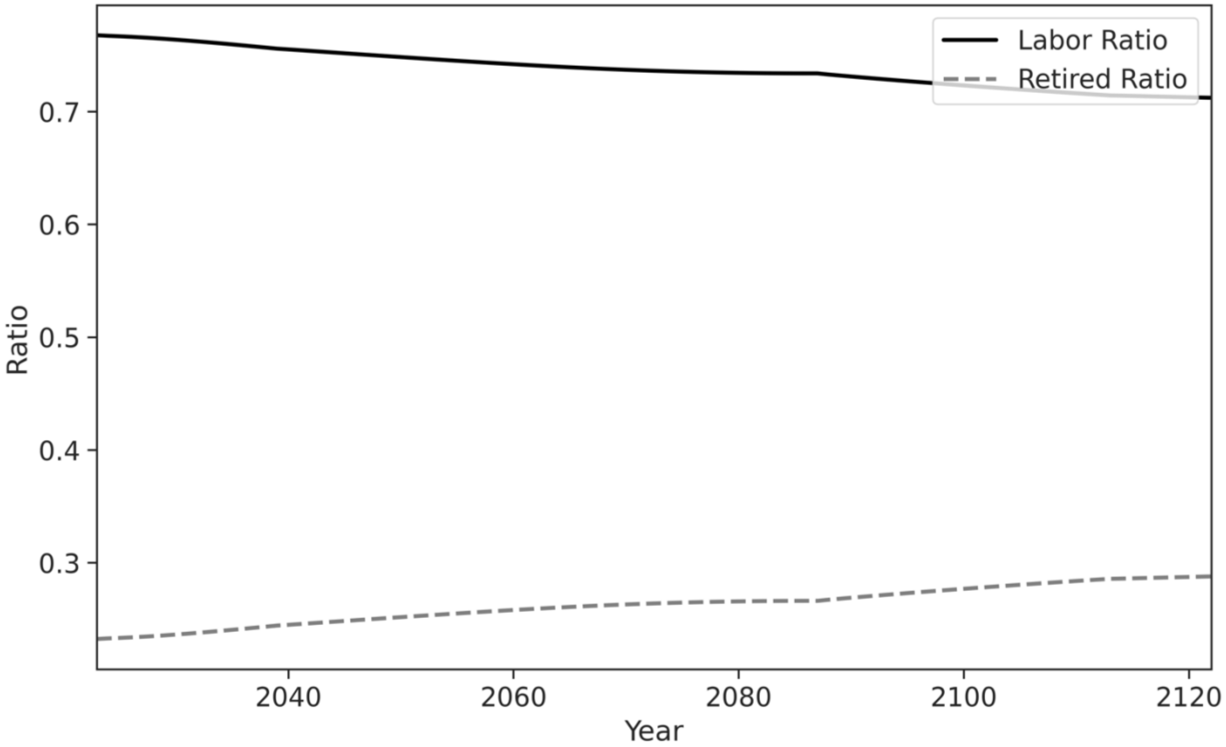


Data Source: Own Presentation

5.3.3. High Birth Rate

Figure 5.9 offers a forward-looking perspective on the labor and retired ratios in Poland over the next century, within the context of high birth rate conditions. This projection reveals a consistent pattern where the labor ratio notably surpasses the retired ratio throughout the entire period. While the labor ratio exhibits a slight downward trend, it remains substantially high, hovering around or above 0.7, indicating a robust working-age population relative to the size of the retired population. Conversely, the retired ratio shows an inverse trend, gradually increasing yet remaining significantly lower compared to the labor ratio. This distinct gap between the labor and retired ratios under high birth rate conditions is markedly larger than what is observed in scenarios with a moderate birth rate, underscoring the demographic advantage of a higher birth rate in sustaining a larger workforce relative to the retired population.

Figure 5.9. Poland-Projections of the Labor Ratio and Retired Ratio over the Next 100 Years under Conditions of High Birth Rate



Data Source: Own Presentation

In Table 5.3, detailing Poland's optimal pension schemes under high birth rate conditions. The DC scheme exhibits the highest target contribution rate at 0.2843, contrasting with the DB scheme's more conservative approach at 0.1528, while the Hybrid scheme takes a moderate stance at 0.1961. The CEC values, indicating the overall welfare participants can expect from each scheme, show the Hybrid scheme as providing the highest welfare at 0.8131, closely followed by the DC scheme at 0.7843, and the DB scheme at 0.7793. Despite these strategic variations, the transition from moderate to high birth rate conditions results in only modest increases in CEC values across all schemes, suggesting that while demographic shifts can influence pension scheme outcomes, the impact on overall welfare is relatively limited. This indicates that the schemes' internal mechanisms, including their responses to demographic changes through adjustments in contribution rates, benefit rates, and investment strategies, might have diminishing returns, underscoring the complex relationship between demographic trends and pension system efficacy.

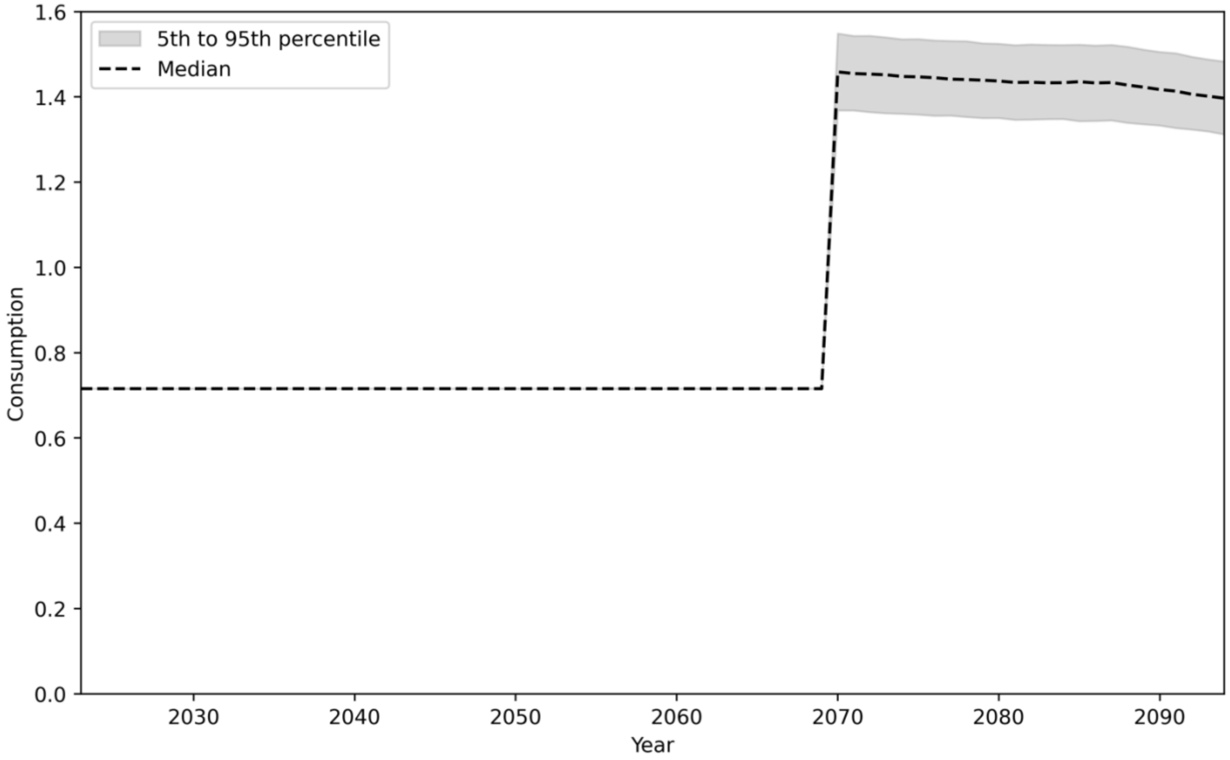
Table 5.3. Poland - Optimal Pension Schemes under Conditions of High Birth Rate

Pension Scheme	DC	DB	Hybrid
p	0.2843	0.1528	0.1961
α	-	0.0274	0.0150
β	0.4607	-	0.0100
ω	0.0250	0.6252	0.5407
b	1.2857	0.7644	0.9362
CEC	0.7843	0.7793	0.8131

Data Source: Own Presentation

Figure 5.10 presents the consumption profile within the optimal DC Scheme in Poland under a high birth rate scenario. The profile exhibits an interesting pattern: from 2023 to 2069, the consumption levels for the 5%, 50% (median), and 95% quantiles are uniformly constant at 0.72. However, in 2070, there's a dramatic change in the consumption pattern, with a sharp increase in the consumption levels to 1.45 at the median, and this elevated level of consumption maintains a slightly decreasing trend but remains significantly higher than the pre-retirement levels through to 2094. The transition to higher consumption levels post-2070 reflects the payout phase of the pension scheme, where retirees begin to draw down on their accumulated savings. The observed pattern underscores the scheme's ability to support significantly higher consumption rates during retirement, albeit with increased variability among individuals. This change signifies the DC scheme's focus on capital accumulation during the working years and its reliance on investment performance, which can lead to diverse outcomes for retirees based on the returns realized on their individual portfolios.

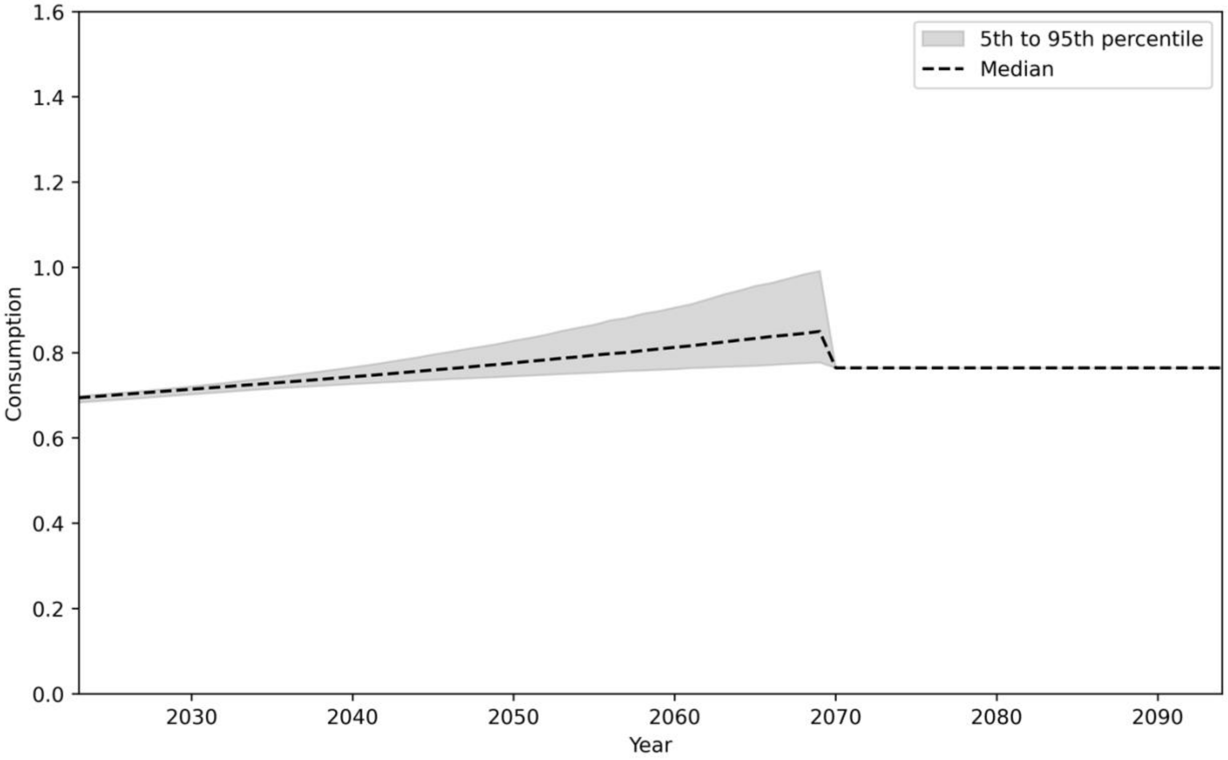
Figure 5.10. Poland - Consumption Profile within Optimal DC Scheme under High Birth Rate Scenario



Data Source: Own Presentation

Figure 5.11 showcases the consumption profile within the optimal DB Scheme in Poland under a high birth rate scenario. The profile demonstrates a gradual and consistent increase in consumption across all quantiles from the beginning of the period until 2069. The median consumption gently rises from 0.69 in 2023 to 0.85in 2069. Notably, in 2070, a pivotal change occurs as the consumption levels for the 5%, 50% (median), and 95% quantiles converge to a uniform figure of 0.76, and this flat rate continues uniformly across all quantiles through to 2094. This dramatic shift to a constant consumption level signifies the DB scheme's structural feature, which ensures a guaranteed, fixed benefit payout upon retirement, regardless of the underlying economic or demographic changes.

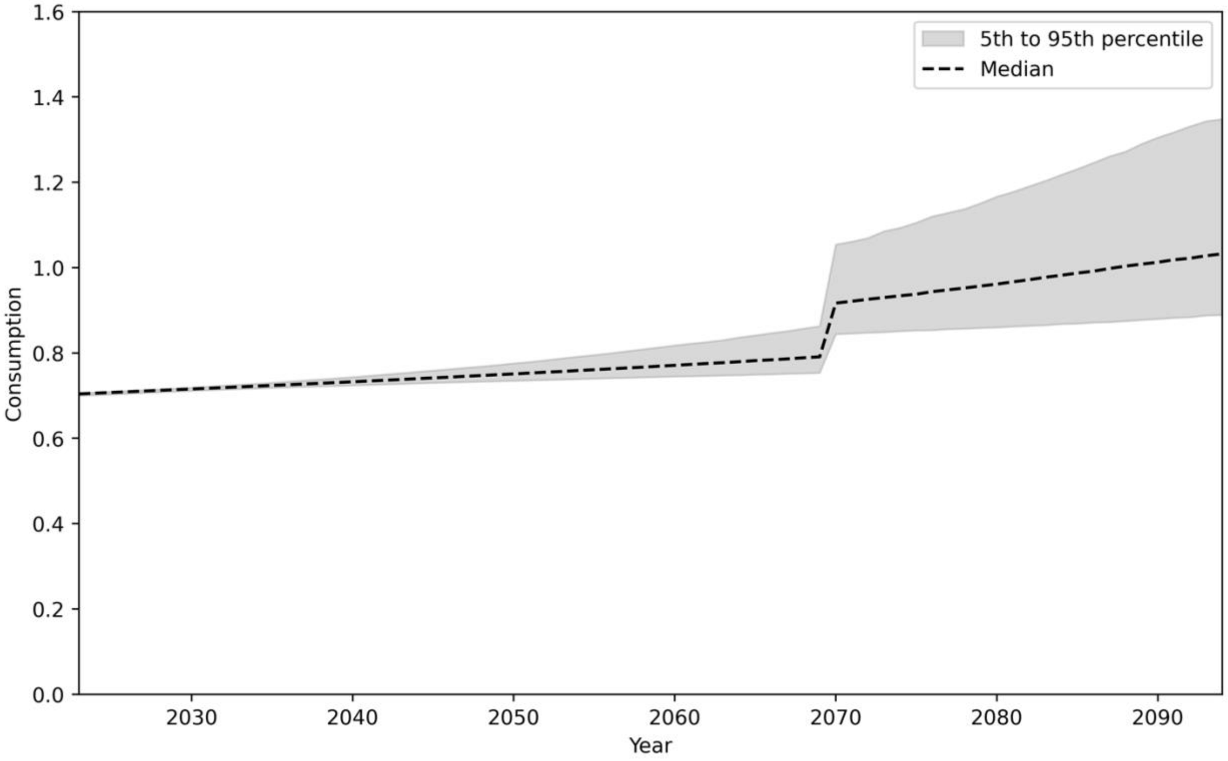
Figure 5.11. Poland - Consumption Profile within Optimal DB Scheme under High Birth Rate Scenario



Data Source: Own Presentation

Figure 5.12 illustrates the consumption profile within the optimal Hybrid Scheme under a high birth rate scenario in Poland, covering the years from 2023 to 2094. This profile showcases a consistent, gradual increase in consumption for the 5%, 50% (median), and 95% quantiles throughout the pre-retirement period, indicating stable and slowly improving living standards. Starting with a median consumption of 0.70 in 2023, the consumption rises steadily to 0.86. In 2070, a significant shift occurs, marking a substantial increase in consumption levels, with the median consumption soaring to 0.92. This remarkable increase represents a transition to retirement, where the scheme's benefits become fully available to the retirees, significantly elevating their consumption capacity. From 2070 onwards, the consumption levels continue to rise, reaching a median of 1.03 by 2094.

Figure 5.12. Poland - Consumption Profile within Optimal Hybrid Scheme under High Birth Rate Scenario



Data Source: Own Presentation

5.4. Robustness Checks

To enhance the reliability and relevance of our findings, a comprehensive series of robustness checks was conducted, focusing on various elements of pension scheme design and their effects under scenarios characterized by high-risk investment environments. These checks were crucial in understanding how different pension schemes—DC, DB, and Hybrid—respond to changes in investment strategies, particularly with an increased allocation towards risky assets.

5.4.1. Adjustment of Investment Strategies

A pivotal part of our robustness analysis involved adjusting the investment parameter omega (ω) to a fixed value of 0.8. This adjustment signifies pushing the allocation towards risky assets to its highest permissible limit within the pension funds, reflecting an aggressive investment stance. As a result, the original optimal parameters set, $\{p, \alpha, \beta, \omega\}$, is modified to exclude ω, thereby recalibrating the analysis to focus on $\{p, \alpha, \beta\}$ under this new investment framework. The outcomes of this robustness check, segmented by birth rate scenarios—low, moderate, and high—reveal insightful trends. Under low birth rate conditions, the recalibrated parameters showed DC schemes at 0.5441, DB schemes at 0.5970, and Hybrid schemes achieving 0.6812. For moderate birth rate scenarios, the values slightly increased with DC at 0.6383, DB at 0.6373, and Hybrid at 0.7387. The most significant adjustments were observed in high birth rate environments, where DC schemes reached 0.7247, DB schemes 0.6961, and Hybrid schemes peaked at 0.7895. These findings underscore the differential impact of shifting towards a high-risk investment strategy across various pension schemes and demographic scenarios. Specifically, the Hybrid pension scheme consistently demonstrated the highest adaptability and performance across all birth rate conditions, suggesting its superior capacity to leverage riskier investment avenues effectively.

Table 5.4. Poland - Adjusted Optimal Parameters for Pension Schemes in a High-Risk Investment Environment

Pension Scheme	DC	DB	Hybrid
Low birth rate	0.5441	0.5970	0.6812
Moderate birth rate	0.6383	0.6373	0.7387
High Birth Rate	0.7247	0.6961	0.7895

Data Source: Own Presentation

5.4.2. Delay Retirement

Table 5.5 provides an insightful analysis into the effects of extending Poland's retirement age from 65 to 67 on the Certainty Equivalent Consumption (CEC) values across different pension schemes (DC, DB, Hybrid) under varying birth rate scenarios (low, moderate, high). This robustness check aligns with international trends observed in countries like Australia, Denmark, Greece, Iceland, Italy, Netherlands, and the US, where retirement ages have been adjusted to reflect longer life expectancies and changing economic conditions (Trading Economics, 2024)

The adjustment of the retirement age to 67 in Poland yields significant enhancements in the CEC values for all pension schemes under varying birth rate scenarios, underscoring the tangible benefits of extending working life on pension scheme outcomes. In the context of a low birth rate, the progression of CEC values is notable, with the Defined Contribution (DC) scheme experiencing an increase to 0.7246, the Defined Benefit (DB) scheme to 0.7600, and the Hybrid scheme achieving an impressive 0.8103. This upward trajectory in welfare measures is even more pronounced in moderate and high birth rate environments. Specifically, in the moderate scenario, CEC values escalate to 0.7810 for DC, 0.7737 for DB, and 0.8289 for Hybrid, while in the high birth rate scenario, they peak at 0.8395 for DC, 0.7863 for DB, and an exceptional 0.8600 for the Hybrid scheme. The comparative analysis reveals that extending the retirement age not only enhances the financial viability of pension schemes but also significantly improves the welfare of participants, as reflected by the higher CEC values. Among the schemes, the Hybrid model consistently outperforms its counterparts across all birth rate scenarios when the retirement age is extended to 67, highlighting its robustness and superior design in maximizing participant welfare under extended working conditions.

Table 5.5. Poland - Optimal Pension Schemes CEC with Retirement Age of 67

Pension Scheme	DC	DB	Hybrid
Low birth rate	0.7246	0.7600	0.8103
Moderate birth rate	0.7810	0.7737	0.8289
High Birth Rate	0.8395	0.7863	0.8600

Data Source: Own Presentation

5.5. Discussion and Conclusion

The comprehensive analysis presented in this study, particularly in the context of Poland's pension system, reveals significant insights into the adaptability and performance of various pension schemes under different demographic scenarios and policy adjustments. This discussion synthesizes the key findings and implications drawn from the simulation results, robustness checks, and the comparative analysis between extending retirement ages within the frameworks of Defined Contribution (DC), Defined Benefit (DB), and Hybrid pension schemes.

The simulation results underscore the sensitivity of pension schemes to underlying demographic trends. Specifically, higher birth rates significantly improve the CEC across all pension schemes, indicating that a younger population structure can bolster the sustainability and welfare outcomes of pension systems. This observation aligns with the findings of Tyrowicz et al. (2018), who note that increased fertility primarily contributes to a higher growth rate of pensions. Similarly, Makarski et al. (2019) observe that higher fertility rates could enhance the solvency of pension systems, reflecting the positive fiscal impact of a younger demographic. However, the magnitude of these improvements varies by scheme, with the Hybrid scheme consistently outperforming the DC and DB models in terms of providing higher welfare to its participants across all demographic scenarios.

Further analysis into the impact of investment strategy adjustments—specifically, setting the investment parameter ω to a high-risk level of 0.8—reveals nuanced insights into how different schemes respond to investment risk. The Hybrid scheme, in particular, exhibits remarkable performance under this high-risk investment strategy across various birth rate conditions. This performance not only highlights the scheme's adeptness at navigating economic volatility but also its capacity to effectively balance risk and reward, thereby ensuring robust welfare outcomes for its participants.

Moreover, the extension of the retirement age from 65 to 67 emerges as a pivotal factor in improving pension schemes' CEC values, aligning with global trends towards delaying retirement. This policy adjustment significantly boosts the welfare outcomes for all schemes, with the Hybrid scheme again showcasing the greatest gains. Such findings underscore the myriad benefits of extending working lives, not just for the financial viability of pension systems but also for the enhanced welfare of future retirees. This aligns with a broader understanding of how pension systems can adapt to and thrive within the context of changing demographic landscapes and economic conditions, with the Hybrid pension scheme exemplified as a particularly resilient and adaptable model.

Both Poland’s pension system policies and country’s demographic is different from China. For example, regarding of pension policies: the contribution rate from employers, enter age, retirement age and investment strategies. Also, although both countries both suffer from the longevity risk, however, Poland’s population structure is different from China. According to the projection, the demographic situation in Poland is better than in China, and the projected labour ratio is overall higher than the Chinese labour ratio under different assumptions of the birth rate.

However, as the employers contribution rate in Poland is much lower than in China, therefore, the overall CEC under different situation is lower than in China.

The comparative analysis between Poland and China's pension systems reveals distinct differences in policies and demographic structures, impacting the sustainability and efficacy of their respective pension schemes. Poland's approach to pension policies diverges notably from China in several key aspects, including employer contribution rates, entry age into the workforce, retirement age, and investment strategies. For instance, Poland's employer contribution rate is significantly lower than China's, which directly influences the overall welfare outcomes for pension participants under various scenarios.

Despite facing similar challenges related to longevity risk, the demographic outlook of the two countries presents contrasting pictures. Poland's demographic projections suggest a relatively healthier population structure compared to China, with a consistently higher labor ratio under different birth rate assumptions. This demographic advantage suggests that Poland's workforce is relatively larger and more sustained over time, potentially offering a more robust base for pension contributions and support. As Bishnu et al. (2019) point out, integrating differential mortality into the model can significantly alter the welfare implications of social security programs. Similarly, Sanchez-Romero et al. (2020) highlight that variations in mortality rates and the application of specific life tables can profoundly affect these outcomes. Their findings suggest that pension systems may need to be specifically tailored to address these demographic variances, thereby enhancing both fairness and efficacy in the system.

However, the lower employer contribution rates in Poland, compared to those in China, result in generally lower CEC values across different scenarios. This indicates that despite having a demographic advantage, the financial underpinning of Poland's pension system—owing to policy settings—does not translate into higher welfare outcomes for its retirees as effectively as it might. Therefore, while Poland benefits from a favorable labor ratio, the impact of policy differences, particularly in terms of employer contributions, underscores the complex interplay between demographic trends and policy frameworks in shaping the success and resilience of national pension systems.

Conclusion

In this dissertation, we have explored the efficacy of hybrid pension schemes in managing demographic risks and maintaining financial sustainability amid aging populations. This research aimed to evaluate and optimize hybrid pension models by using real demographic data from China and Poland, assessing their performance in comparison to traditional DB and DC schemes under various demographic and economic conditions. This investigation is driven by the global challenge of demographic shifts, such as increased life expectancy and declining birth rates, which pose significant concerns for the viability and equity of pension systems. By comparing the outcomes of the proposed hybrid model with those of DB and DC systems, this study seeks to demonstrate the potential advantages of hybrid schemes in enhancing the adaptability and effectiveness of pension plans in response to these demographic challenges.

This research has provided a comprehensive analysis of how demographic trends, such as birth rates, significantly influence pension schemes, highlighting that higher birth rates consistently enhance the CEC across all models. This suggests that a younger population structure can substantially improve the sustainability and welfare outcomes of pension systems. Notably, the proposed Hybrid pension scheme excels in delivering superior welfare outcomes compared to the DC and DB models across various demographic scenarios. This superiority is attributed to the Hybrid scheme's flexibility and resilience, enabling it to dynamically adapt to demographic and economic fluctuations effectively. These findings provide strong support for Hypothesis 1 (H1), which posits that hybrid pension schemes offer greater stability and flexibility compared to traditional pension models when confronted with demographic shifts and economic uncertainties.

Furthermore, the study delves into the impact of investment strategies and adjustments in retirement age on pension schemes. Observations indicate that even with a higher allocation towards risky assets, the Hybrid scheme can remain robust, maintaining outstanding performance in these high-risk investment environments. Additionally, extending the retirement age has been shown to significantly bolster the financial viability and welfare outcomes of all pension schemes, including DB, DC, and Hybrid models. Notably, even in these scenarios, the Hybrid pension scheme consistently demonstrates the best performance. These observations affirm Hypothesis 2 (H2), indicating that hybrid pension schemes demonstrate superior performance in terms of risk management and sustainability, particularly in contexts characterized by varying birth rates and aging populations

In a comparative analysis of the pension systems in China and Poland, the study offers an insightful look into how unique demographic and policy contexts affect pension outcomes in each country. China is grappling with significant demographic challenges due to its aging population, driven by its long-standing one-child policies and increasing life expectancy. This shift places considerable strain on the pension system as the ratio of retirees to working-age individuals continues to grow. In such a context, the Hybrid pension scheme consistently provides the best outcomes in terms of CEC, offering superior welfare benefits compared to the DC and DB schemes, and showing robustness under aggressive investment strategies.

Conversely, Poland presents a scenario with less demographic strain, thanks to a healthier population structure and a higher labor ratio, which provides a solid foundation for pension contributions and system support. However, despite these advantages, the overall CEC values in Poland are generally lower than those observed in China, likely due to different economic conditions and policy settings, such as lower employer contribution rates. This indicates that while Poland benefits from favorable demographics, its pension system design may not fully leverage this advantage to enhance welfare outcomes. These findings support Hypothesis 3 (H3), which posits that there are significant differences in the suitability of pension models across different national contexts, influenced by unique demographic structures and policy frameworks. Furthermore, the Hybrid pension scheme is still identified as potentially the optimal solution for Poland, given its capacity to adapt to diverse demographic and economic conditions.

The study also highlights that extending the retirement age could significantly enhance the sustainability of pension schemes, with increased CEC values across all models suggesting improved welfare for retirees. The effectiveness of such policy adjustments varies, reflecting the specific economic and demographic conditions of each country.

Policy recommendations arising from this study emphasize the importance of adopting hybrid pension schemes, especially in countries facing demographic pressures similar to those observed in China and Poland. Such schemes are not only adept at balancing risk but also enhance the predictiveness and adequacy of retirement benefits. This dissertation advocates for policies that support the transition towards hybrid systems, including regulatory frameworks that facilitate risk-sharing and ensure the equitable distribution of pension benefits across generations. Given the beneficial impacts of extending working life shown in the research, it is recommended that countries consider policies to delay retirement age. This adjustment could significantly improve the financial viability of pension systems, better aligning them with increased life expectancies and promoting longer periods of economic contribution by the working population.

For future academic research, numerous areas could deepen our understanding of hybrid pension schemes and their efficacy. Investigating the actuarial and financial mechanisms within hybrid pensions is crucial. Such research would delve into how these schemes distribute risks and rewards between employers and employees, examining their resilience under various economic conditions. This could include rigorous testing of the actuarial assumptions and financial models that underpin these plans to predict their performance in different market environments better.

Incorporating advanced predictive analytics and artificial intelligence (AI) in pension management presents another promising research direction. These technologies have the potential to significantly enhance the precision of demographic and financial forecasts, leading to more adaptive and robust pension systems. Studies could explore the integration of AI tools for real-time data analysis and decision-making processes within pension fund management, aiming to optimize asset allocation and mitigate risks more effectively.

Additionally, understanding cultural attitudes towards savings and retirement is vital. It's important to examine how societal views on intergenerational support and retirement planning influence the adoption and success of pension schemes. Research in this area could provide valuable insights into designing pension systems that are not only financially sound but also culturally aligned with the populations they serve.

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Appendix

Appendix 1. Partial Code for Chapter 2: Lee-Carter model

```
import pandas as pd
import numpy as np
from sklearn.decomposition import PCA
import pmdarima as pm
import matplotlib.pyplot as plt



# Transpose and set 'Age' as index
mortality_data = pd.read_excel("China_Mortality.xlsx")
mortality_data_transposed = mortality_data.set_index('Age').transpose()

# Convert both column names and index to string to ensure compatibility
mortality_data_transposed.columns = mortality_data_transposed.columns.astype(str)
mortality_data_transposed.index = mortality_data_transposed.index.astype(str)

# Split the data into training and testing sets
train_data = mortality_data_transposed.loc['1997':'2017']
test_data = mortality_data_transposed.loc['2018':'2022']

# Log transformation
log_train_data = np.log(train_data)

# Apply PCA to the training set
pca = PCA(n_components=1)
k_train = pca.fit_transform(log_train_data).flatten()
a = log_train_data.mean(axis=0).values
b = pca.components_[0]
```

```
'''
ARIMA Model Fitting: The code fits an ARIMA model to the transformed mortality data to forecast the principal component scores ('k_t'), configuring the model to simulate a random walk with a drift component.

Forecast Generation: It then uses the fitted model to predict 'k_t' for future periods corresponding to the testing data years.

'''

predicted_mortality_df = pd.DataFrame(predicted_mortality_rates,
                                      index=test_data.index,
                                      columns=test_data.columns)

# Compute RMSFE
actual_mortality_rates = np.log(test_data).values
rmsfe = np.sqrt(np.mean((actual_mortality_rates - predicted_mortality_rates) ** 2))

print("RMSFE:", rmsfe)
```

Appendix 2. Partial Code for Chapter 2: LC-G

```
import pandas as pd
import numpy as np
from sklearn.decomposition import PCA
import pmdarima as pm
from scipy.optimize import minimize
import matplotlib.pyplot as plt

# Load and preprocess the mortality data
mortality_data = pd.read_excel("China_Mortality.xlsx")
mortality_data_transposed = mortality_data.set_index('Age').transpose()
mortality_data_transposed.columns = mortality_data_transposed.columns.astype(str)
mortality_data_transposed.index = mortality_data_transposed.index.astype(str)

# Log transformation of the training and testing data
log_train_data = np.log(mortality_data_transposed.loc['1997':'2017'])
log_test_data = np.log(mortality_data_transposed.loc['2018':'2022'])

# Apply PCA to extract initial parameters
pca = PCA(n_components=1)
k_train = pca.fit_transform(log_train_data).flatten()
a = log_train_data.mean(axis=0).values
b_initial = pca.components_[0]

'''
Define and utilize advanced statistical functions to optimize the model parameters
and compute RMSFE using a custom objective function linked to age-specific mortality rates.
'''

# Define the Inversed Epanechnikov Kernel Function
def inv_epanechnikov_kernel(tau, beta):
```

```
    pass

# Function to compute RMSFE for given r_base and beta
def compute_rmsfe(params):
    pass

# Grid search for optimal r_base and beta
optimal_params = minimize()
optimal_r_base, optimal_beta = optimal_params
print("Optimal r_base:", optimal_r_base, "Optimal beta:", optimal_beta)
```

Appendix 3. Partial Code for Chapter 2: LC-H

```
import pandas as pd
import numpy as np
from sklearn.decomposition import PCA
import pmdarima as pm
from scipy.optimize import minimize
import matplotlib.pyplot as plt

# Load and preprocess the mortality data
mortality_data = pd.read_excel("China_Mortality.xlsx")
mortality_data_transposed = mortality_data.set_index('Age').transpose()
mortality_data_transposed.columns = mortality_data_transposed.columns.astype(str)
mortality_data_transposed.index = mortality_data_transposed.index.astype(str)

# Log transformation of the training data
log_train_data = np.log(mortality_data_transposed.loc['1997':'2017'])
log_test_data = np.log(mortality_data_transposed.loc['2018':'2021'])

# Apply PCA to extract initial b_x and k_t values
```

```
pca = PCA(n_components=1)
k_train = pca.fit_transform(log_train_data).flatten()
a = log_train_data.mean(axis=0).values
b_initial = pca.components_[0]

# Define the Inversed Epanechnikov Kernel Function
def inv_epanechnikov_kernel(tau, beta):
    pass  # Computation details hidden for confidentiality

def lambda_h(h, delta_x):
    pass  # Recursive computation details hidden for confidentiality

# Function to compute RMSFE for given delta_1 and beta
def compute_rmsfe(params):
    pass  # Computation details hidden for confidentiality
optimal_params = minimize()
optimal_delta_1, optimal_beta = optimal_params
print("Optimal delta_1:", optimal_delta_1, "Optimal beta:", optimal_beta)
```

Appendix 4. Partial Code for Chapter 2: Forecasting for the final model

```
import pandas as pd
import numpy as np
from sklearn.decomposition import PCA
import pmdarima as pm
from scipy.optimize import minimize

# Load and preprocess the mortality data
mortality_data = pd.read_excel("China_Mortality.xlsx")
mortality_data_transposed = mortality_data.set_index('Age').transpose()
mortality_data_transposed.columns = mortality_data_transposed.columns.astype(str)

# Log transformation of the data
log_mortality_data = np.log(mortality_data_transposed)

# Apply PCA to extract initial b_x and k_t values
pca = PCA(n_components=1)
k_all = pca.fit_transform(log_mortality_data).flatten()
a = log_mortality_data.mean(axis=0).values
b_initial = pca.components_[0]
# Define the Inversed Epanechnikov Kernel Function and hyperbolic decay function
def inv_epanechnikov_kernel(tau, beta):
    pass

def lambda_h(h, delta_x):
    pass

# Function to generate forecasts for mortality rates for 65 years
def generate_forecast(params):
    pass
```

```
# Grid search for optimal delta_1 and beta
optimal_params = minimize()
optimal_delta_1, optimal_beta = optimal_params
print("Optimal delta_1:", optimal_delta_1, "Optimal beta:", optimal_beta)
# Generate forecast using optimal delta_1 and beta
optimal_forecasts = generate_forecast(optimal_params)

# Convert forecasted mortality rates to DataFrame
forecast_years = range(2023, 2023 + 100)
forecasted_df = pd.DataFrame(optimal_forecasts, index=forecast_years, columns=mortality_data_transposed.columns)

# Save the forecasted data to an Excel file
excel_output_path = "China_forecasted_mortality_rates_LC-H.xlsx"
forecasted_df.to_excel(excel_output_path)

print(f"Forecasted mortality rates saved to {excel_output_path}")
```

Appendix 5. Partial Code for Chapter 2: Population

```
import pandas as pd
import matplotlib.pyplot as plt

# Load the mortality rates data, with forecasts starting from 2023
mortality_rates_data = pd.read_excel('China_forecasted_mortality_rates_LC-H.xlsx')
mortality_rates_data.set_index('Unnamed: 0', inplace=True)
mortality_rates_data.index = mortality_rates_data.index.astype(int)
mortality_rates_data.columns = mortality_rates_data.columns.astype(int)
mortality_rates_data /= 100  # Convert percentages to proportions

# Calculate survival probabilities
survival_probabilities = 1 - mortality_rates_data

# Load the initial population distribution for 2022
population_data = pd.read_excel('China_Polulation_by_Age.xlsx')
# Set the MAX_AGE based on available mortality rates data
MAX_AGE = 90
# Define birth rate scenarios
birth_rate_scenarios = [i/1000 for i in range(6, 17, 2)]  # From 6/1000 to 16/1000 with a step of 2/1000
# Line styles for black and white plot
line_styles = ['-', '--', '-.', ':', (0, (3, 5, 1, 5)), (0, (3, 1, 1, 1))]
# Plotting setup
plt.figure(figsize=(10, 6))
for i, birth_rate in enumerate(birth_rate_scenarios):
# Create a DataFrame for the projected population for each scenario
# Adjust the population projection loop for each birth rate scenario
# Calculate the total population for each year for the current scenario
# Normalize the total population so that 2022's population is 1
    pass
```

```
plt.plot(normalized_population_by_year.index, normalized_population_by_year, color='black', linestyle=line_styles[i % len(line_styles)], label=f' {birth_rate*1000}')

# Finalize the plot
plt.title('Normalized Projected Total Population by Birth Rate Scenario (2022=1)')
plt.xlabel('Year', fontsize=14)
plt.ylabel('Normalized Total Population', fontsize=14)
plt.legend()
plt.savefig("China_Normalized_Projected_Population_Different_Birth_Rate.png", dpi=600, bbox_inches='tight')
```

Appendix 6. Partial Code for Chapter 2: Population Proportion

```
import pandas as pd
import matplotlib.pyplot as plt

# Function to calculate age group proportions
def calculate_age_group_proportions(projected_population):
    age_group_proportions = pd.DataFrame(index=projected_population.columns, columns=['0-24', '25-64', '65-90'])
    for year in projected_population.columns:
        total_population = projected_population[year].sum()
        age_group_proportions.at[year, '0-24'] = projected_population.loc[projected_population.index.astype(int) <= 24, year].sum() / total_population
        age_group_proportions.at[year, '25-64'] = projected_population.loc[(projected_population.index.astype(int) >= 25) & (projected_population.index.astype(int) <= 64), year].sum() / total_population
        age_group_proportions.at[year, '65-90'] = projected_population.loc[(projected_population.index.astype(int) >= 65) & (projected_population.index.astype(int) <= 90), year].sum() / total_population
    return age_group_proportions

# Function to plot age group proportions with style consistent with the mortality rate projections
def plot_age_group_proportions(ax, age_group_proportions, title):
    line_styles = ['-', '--', '-.', ':', (0, (3, 5, 1, 5)), (0, (3, 1, 1, 1))]
    ax.plot(age_group_proportions.index, age_group_proportions['0-24'], linestyle=line_styles[0], color='black', label='Ages 0-24')
    ax.plot(age_group_proportions.index, age_group_proportions['25-64'], linestyle=line_styles[1], color='black', label='Ages 25-64')
    ax.plot(age_group_proportions.index, age_group_proportions['65-90'], linestyle=line_styles[2], color='black', label='Ages 65-90+')
    ax.set_title(title, fontsize=16)
```

```
    ax.set_xlabel('Year', fontsize=14)
    ax.set_ylabel('Proportion of Total Population', fontsize=14)
    ax.set_xticks(ax.get_xticks()[::20])
    ax.legend(title='Age Groups')

# Load projected population data for China and Poland
china_projected_population = pd.read_csv('China_low_projected_population_2023_to_2122.csv',
index_col=0)
poland_projected_population =
pd.read_csv('Poland_low_projected_population_2023_to_2122.csv', index_col=0)

# Calculate age group proportions for China and Poland
china_age_group_proportions = calculate_age_group_proportions(china_projected_population)
poland_age_group_proportions = calculate_age_group_proportions(poland_projected_population)

# Plotting setup for two subplots
fig, axes = plt.subplots(nrows=1, ncols=2, figsize=(16, 6))

# Plot age group proportions for China and Poland with the specified style
plot_age_group_proportions(axes[0], china_age_group_proportions, 'China')
plot_age_group_proportions(axes[1], poland_age_group_proportions, 'Poland')

plt.savefig("Low_Population_Proportions_by_Age_Group_China_Poland.png", dpi=600,
bbox_inches='tight')
```

Appendix 7. Partial Code for Chapter 4-Hybrid Pension with Moderate Birth Rate

```
import numpy as np
import pandas as pd
from scipy.integrate import quad
from skopt import gp_minimize
from skopt.space import Real
from skopt.utils import use_named_args


# Setting the seed for reproducibility
np.random.seed(0)
# Load population data
population_data = pd.read_csv("China_moderate_projected_population_2023_to_2122.csv", index_col=0)

enter_age = 16
retire_age = 60
retire_plus_age = 90
start_year = 2023
projection_length = 100

T=74
R=44
sigma=0.15
def calculate_population_proportions_for_100_years(df, enter_age, retire_age, retire_plus_age, start_year, projection_length=100):
    pass
```

```
# Calculate labor and retirement ratios
labor_ratios, retirement_ratials =
calculate_population_proportions_for_100_years(population_data, enter_age, retire_age,
retire_plus_age, start_year, projection_length)

def calculate_liability_for_cohort(x, b, p, r, R, T):
    pass

def calculate_L_dynamic(b, p, r, retire_plus_age, retire_age, labor_ratios, retirement_ratials):
    pass

def simulate_collective_scheme(p, α, β, omega, labor_ratios, retirement_ratials, T=74, R=44,
gamma=5, delta=0.04, r=0.02, mu=0.06, J=5000):
    pass
# Define the parameter space for Bayesian Optimization
space = [
    Real(0.00, 1, name='p'),
    Real(0.00, 1, name='alpha'),
    Real(0.00, 1, name='beta'),
    Real(0.00, 0.3, name='omega')
]
# The objective function that Bayesian Optimization will minimize
# Adjusted Objective Function with Penalty Consideration
@use_named_args(space)
def objective(p, alpha, beta, omega):
    # Check if the constraint is satisfied
    is_constraint_satisfied = (alpha + beta) > 0.02

    # Calculate the actual objective value
    objective_value = -simulate_collective_scheme(p, alpha, beta, omega, labor_ratios,
retirement_ratials)
```

```
    # Apply penalty if the constraint is not satisfied
    if not is_constraint_satisfied:
        penalty = 1e+6  # Consider adjusting the penalty magnitude based on the scale of objective values
        objective_value += penalty

    return objective_value
# Run Bayesian Optimization
res_gp = gp_minimize(objective, space, n_calls=100, random_state=0)
# Results
print("Best score:", -res_gp.fun)
print("Best parameters:\np: {}\nalpha: {}\nbeta: {}\nomega: {}".format(*res_gp.x))

p_opt, alpha_opt, beta_opt, omega_opt = res_gp.x
# Define a new, narrower space around the optimal parameters
space_refined = [
    Real(max(0.00, p_opt - 0.01), min(1, p_opt + 0.01), name='p'),
    Real(max(0.00, alpha_opt - 0.01), min(1, alpha_opt + 0.01), name='alpha'),
    Real(max(0.00, beta_opt - 0.01), min(1, beta_opt + 0.01), name='beta'),
    Real(max(0.00, omega_opt - 0.01), min(0.3, omega_opt + 0.01), name='omega')
]

# Run Bayesian Optimization again with the refined space
res_gp_refined = gp_minimize(objective, space_refined, n_calls=50, random_state=0)

print("Refined BO Best score:", -res_gp_refined.fun)
print("Best parameters:\np:{}\nalpha: {}\nbeta: {}\nomega: {}".format(*res_gp_refined.x))
CEC = np.power(-res_gp_refined.fun * (1 - gamma) * delta / (1 - np.exp(-delta * T)), 1 / (1 - gamma))
print("Calculated Egg Crate Welfare Equivalent Cachet [Popular Term CEC]:", CEC)
```

Summary

This dissertation proposes a novel hybrid pension scheme that incorporates a demographic model to address the sustainability challenges faced by pension systems due to demographic shifts and economic uncertainties. This study calibrates the theoretical model within the specific contexts of China and Poland, examining how integrating elements of Defined Benefit (DB) and Defined Contribution (DC) plans into a hybrid model can enhance stability and intergenerational equity. The core methodology employs the Lee-Carter model, modified with geometric and hyperbolic adjustments, to accurately forecast mortality rates and demographic changes. Additionally, the Overlapping Generations (OLG) model is utilized to explore economic interactions and their impact on pension outcomes. The analysis employs Monte Carlo simulations and Bayesian Optimization to assess and optimize the hybrid scheme's performance across various economic scenarios, focusing on Certainty Equivalent Consumption (CEC) as a welfare measure. Results indicate that the proposed hybrid pension scheme offers greater resilience and adaptability than traditional pension models, particularly in managing demographic risks and economic fluctuations. Policy recommendations suggest adopting hybrid pension schemes, especially in countries with demographic profiles similar to China and Poland. The study advocates for policies that support the transition towards such schemes, including delayed retirement to improve the financial health of pension systems. This dissertation contributes significant new insights into pension economics, offering robust solutions for enhancing pension system sustainability in the face of global aging trends. Future academic research should focus on investigating the actuarial and financial mechanisms of hybrid pension schemes, utilizing advanced predictive analytics and AI to enhance management precision, and exploring how cultural attitudes towards savings and retirement affect pension system adoption and efficacy.

Streszczenie

Niniejsza rozprawa proponuje nowy, hybrydowy system emerytalny, który włącza model demograficzny w celu rozwiązania problemów zrównoważenia systemów emerytalnych wynikających ze zmian demograficznych i niepewności ekonomicznych. Badanie to kalibruje teoretyczny model w konkretnych kontekstach Chin i Polski, analizując, jak integracja elementów systemów Defined Benefit (DB) i Defined Contribution (DC) w model hybrydowy może zwiększyć stabilność i równość międzypokoleniową. Podstawowa metodologia wykorzystuje model Lee-Cartera, zmodyfikowany o korekty geometryczne i hiperboliczne, aby dokładnie prognozować wskaźniki śmiertelności i zmiany demograficzne. Ponadto, model pokoleń nakładających się (OLG) jest wykorzystywany do badania interakcji ekonomicznych i ich wpływu na wyniki systemów emerytalnych. Analiza korzysta z symulacji Monte Carlo i optymalizacji bayesowskiej, aby ocenić i zoptymalizować wydajność hybrydowego schematu w różnych scenariuszach ekonomicznych, koncentrując się na równoważnym spożyciu (CEC) jako miarze dobrobytu. Wyniki wskazują, że proponowany hybrydowy system emerytalny oferuje większą odporność i adaptacyjność niż tradycyjne modele emerytalne, szczególnie w zarządzaniu ryzykiem demograficznym i fluktuacjami ekonomicznymi. Zalecenia polityczne sugerują przyjęcie hybrydowych systemów emerytalnych, zwłaszcza w krajach o profilach demograficznych podobnych do Chin i Polski. Badanie opowiada się za politykami wspierającymi przejście na takie systemy, w tym opóźnienie wieku emerytalnego w celu poprawy kondycji finansowej systemów emerytalnych. Niniejsza rozprawa wnosi istotne nowe spostrzeżenia do ekonomii emerytalnej, oferując solidne rozwiązania dla zwiększenia zrównoważenia systemów emerytalnych w obliczu globalnych trendów starzenia się. Przyszłe badania naukowe powinny koncentrować się na badaniu mechanizmów aktuarialnych i finansowych hybrydowych systemów emerytalnych, wykorzystaniu zaawansowanej analizy predykcyjnej i sztucznej inteligencji do zwiększenia precyzji zarządzania, oraz na badaniu, jak kulturowe postawy wobec oszczędzania i emerytury wpływają na przyjęcie i skuteczność systemów emerytalnych.